

Buyer-Initiated Auctions: Nominations in Oil & Gas

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Abstract

Auctions fail when too few bidders participate. In oil and gas leasing, a common remedy is pre-auction nomination: a tract is offered only if some bidder pays a non-refundable fee and commits to buying at the minimum bid. This paper asks how nomination compares with the more traditional auction design where it is the seller who chooses what to offer. I develop a two-stage nomination-auction game in which bidders' values are correlated but costly to learn. Upon arriving during the nomination stage, bidders choose whether to learn early and nominate or to wait and learn only about tracts that others have nominated. Using novel data from North Dakota oil and gas lease sales, I find nomination raised seller revenue by about \$3.05 million (44%) across the 728 tracts in the 2017–2018 estimation sample, relative to a seller who chooses arbitrarily which tracts to offer. Nomination is valuable mainly because it guarantees a bidder when attention is scarce: every nominated tract sells, while when tracts are offered without nomination, about 90 percent of them sell. Also, that extra guaranteed bidder raises the average winning bid by about 30 percent through the competition it adds. The gains accrue almost entirely to the seller.

1 Introduction

Auctions are a primary mechanism for allocating resources, from private goods to government assets. Before any auction can be designed, however, the seller must decide which items to

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offer. When running each auction is costly, that decision matters: an item that draws no bidder still has to be prepared and offered. And when demand is uncertain, the seller cannot tell in advance which items those are. The problem is illustrated by recent U.S. government attempts to auction oil and gas leases in Alaska’s Arctic National Wildlife Refuge, where the January 2025 Bureau of Land Management (BLM) lease sale offered about 400,000 acres and received zero bids.¹ When demand for an item may not exist at all, the standard design instruments—reserve prices and entry fees—cannot help, because they act only on bidders who have already shown up.

This paper studies a mechanism built for exactly this problem: pre-auction nomination. Instead of having the seller decide which items to auction, it is the buyers who nominate the items they wish to see offered. Nomination entails paying a small fee and committing to purchase the item at a minimum price (the reserve price) if no other bidder appears. Only items that receive a nomination proceed to auction.² Nomination is widely used in mineral leasing: between 2009 and 2019 about 87 million acres were nominated for federal onshore oil and gas leasing [U.S. Government Accountability Office, 2021, p. 18], many U.S. states lease their mineral rights this way, and a similar system determined which Outer Continental Shelf tracts the federal government offered before area-wide leasing was adopted in 1983 [Hendricks and Porter, 2014].

I ask what nomination is worth, and to whom. I compare nomination used in practice against a benchmark in which the seller chooses which tracts to offer without input from buyers, and measure the difference along four outcomes: 1. the seller’s revenue; 2. the *sale rate*, i.e. how often an offered tract draws any bidder at all; 3. the surplus of the bidder who nominates; and 4. the surplus of the bidders who bid without nominating. Nomination can move these outcomes through four channels. First, it supplies a *guaranteed bidder*: every offered tract has at least one bidder present and committed to buying it. Second, *more valuable tracts are offered*: a bidder nominates only after finding the tract valuable enough. Third, nomination *concentrates attention*: bidders with limited attention who cannot monitor thousands of candidate tracts can attend to the short list of nominated

¹For such BLM-run oil and gas lease auctions, environmental reviews required under the National Environmental Policy Act must be completed before the auction, regardless of whether or not those lands are ultimately leased. Across twelve reviews conducted between June 2020 and January 2021, the cost ranged from \$17,000 to \$178,000 per lease sale, and BLM Utah reported an average of about \$2,100 per parcel across its two most recent auctions [U.S. Government Accountability Office, 2021, pp. 28–29, n.42].

²In U.S. federal onshore oil and gas leasing, this same mechanism is officially termed an *expression of interest* (EOI): a member of the public may nominate a parcel for a lease sale by submitting an EOI, and BLM may also select parcels directly [U.S. Government Accountability Office, 2021]. I use *nomination* throughout to match the terminology of my empirical setting (North Dakota state leasing). Section 1.1 situates this mechanism relative to related but distinct notions of “expressions of interest” and “indications of interest” in the finance literature (IPO bookbuilding, M&A sale processes).

ones, so more of them arrive per tract. Fourth, when bidders’ values are correlated but learned only at a cost, a nomination creates an *information spill over*: it tells other bidders that the tract is likely valuable to them too, so more of them pay to learn. The spill over has a countervailing force, *under-provision*: a bidder who can wait to learn from someone else’s nomination has less reason to learn early and nominate herself, pushing less bidders to nominate. This last channel requires correlated values and costly learning while the other three works without these features.

To quantify these channels I build a two-stage nomination-auction game. Potential bidders arrive over time according to two Poisson processes, one before nomination and one after, whose rates are allowed to differ between the two stages. Values are correlated private values: firms draw on the same geology and commodity prices, so their values move together, but each firm’s value depends on its own technology and lease portfolio and can be learned only by paying a privately known learning cost. A bidder arriving before nomination chooses among learning early, waiting to learn only if someone else nominates, and exiting; a bidder arriving after nomination decides whether to learn and enter. The early learning bidder chooses whether to nominate. A tract with at least one nomination is sold by a second-price auction, and the number of nominations is not revealed. In equilibrium, bidders follow cutoff rules: low-cost bidders learn early, intermediate-cost bidders wait, and high-cost bidders exit. Likewise, an early learner nominates if her value exceeds a cutoff, the *nomination threshold*.

I estimate the model with lease auctions run by the State of North Dakota in 2017–2018: 878 auctions and 2,740 bids, combined with nomination records. Estimation proceeds in two steps. First, I recover the value distribution and the correlation level from bids. Because North Dakota uses proxy bidding, the winner pays one increment above the second-highest bid and the winning bid is never observed.³ I develop a copula-based estimator that matches the observed order statistics to their theoretical distribution under exchangeable copula dependence. Second, I estimate the average number of bidders who arrive at a tract in each stage (the rates of the two Poisson arrival processes), along with the learning-cost distribution. Because bidders’ values scale with public tract characteristics (e.g., how close a tract is to an existing well) while the fee, the reserve, and learning costs are fixed in dollars, the equilibrium differs across tracts. I control for this observable, tract-level heterogeneity with a single multiplier on every bidder’s value that summarizes how promising a tract looks. What remains, the bidder’s homogenized value, is private and reflects bidder-level differences

³Under proxy bidding, each bidder submits a maximum willingness to pay (the proxy bid) to the online platform, which then bids on her behalf in minimum increments up to that maximum. The auction therefore works like an ascending button auction played by the computer, and the bidder with the highest proxy bid wins at one increment above the second-highest proxy bid. See Section 2.

that I do not observe, such as drilling technology and lease portfolio. I solve the equilibrium separately for 19 groups of tracts (bins) ordered by this multiplier.

The estimates show that, even after controlling for observable tract characteristics, bidders' values remain strongly correlated ($\hat{\rho} = 0.86$). The mean learning cost is about \$1,161, small against an average bid of about \$11,800, so on the more promising tracts in terms of observables (e.g., close to existing wells or large in size; formally defined in Section 5.1) essentially every firm that considers a tract finds it worth evaluating. I also find that few firms look at any given tract before nomination: the pre-nomination arriving bidder count is about 0.005 per tract on average, against about 2.9 after nomination. This is consistent with a low nomination rate against high auction participation. In each season, the quarterly cycle that runs from one lease sale to the next (Section 2), firms submit about 98 nominations against about 28,000 tracts available for nomination, or $98/28,000 \approx 0.35\%$ nominations per available tract. Yet an auctioned tract draws 3.3 bidders on average.

I then use the estimated model to switch the channels off one at a time—first the information spill over, then the selection of more valuable tracts, then the guaranteed bidder—ending at a benchmark in which the seller offers tracts arbitrarily; the three steps sum to the value of nomination. To measure under-provision, I also remove the information a nomination conveys to other bidders and re-solve the learning-cost cutoff below which bidders learn early.

Because concentrated attention changes how many bidders look at a tract in the first place, I isolate it in a separate one-stage auction game with no nomination stage, in which arriving bidders decide whether to learn and then bid. To set how many bidders arrive at a tract in this market, I estimate an *attention function*: how much response a tract receives in terms of either nominations during the nomination stage or bids during the auction stage. The number of responses falls as the number of tracts that requires bidders' attention grows. I evaluate the one-stage market first at the average arriving bidder count this function implies when attention is spread over all $\approx 28,000$ tracts available for nomination, and then at the count it implies when the same attention is concentrated on the much smaller set of offered tracts (≈ 73 in a typical season).

I find that, across the 728 tracts in the estimation sample, nomination raises seller revenue by about \$3.05 million, or 44% (from \$6.86 million to \$9.91 million), relative to a seller who chooses arbitrarily which tracts to offer.⁴ Almost all of this gain (\$3.01 million) comes from the *guaranteed bidder*: every nominated tract sells, while about 10% of arbitrarily

⁴These totals evaluate each tract at its own observable characteristics (Appendix Table 20). Section 7 evaluates every tract in a bin at the bin's median tract, which gives \$6.40 million and \$9.28 million (Table 3); the split across channels is essentially unchanged.

offered tracts draw no bidder, and the winning bid on a sold tract is about 30% higher because the guaranteed bidder adds competition. The *selection of more valuable tracts* adds essentially nothing. At the estimates, the nomination threshold is barely above the reserve, so almost any early learner whose value is high enough to bid is also willing to pay the fee and nominate. Total surplus rises by about \$3.3 million: nominators gain \$2.1 million, and non-nominators lose \$1.9 million as the guaranteed nominating bidder competes away their rents. The information spill over is small. It raises the share of later arrivals who pay to learn and bid only from 98.0% to 98.1% on average, because almost all of them learn even without the signal, and by 1.4 percentage points on the least promising tracts (the 4% of the 728 estimation tracts in the lowest bins). Consequently, the under-provision it could cause is also negligible: removing the signal changes the probability that an early arrival nominates by only 0.04 percentage points on average, as small decreases in some bins offset small increases in others.

Whether or not a tract sells matters for tracts that look less valuable on observable characteristics, such as being far from existing wells or small: an arbitrarily offered tract of this kind draws a bidder only 1.4% of the time, and nomination shifts its sale rate to one.

Nomination also lowers the learning costs that bidders sink. On the tracts it offers, wasted learning—the cost paid by bidders who learn a value below the reserve and therefore never bid—roughly halves, from about \$272 to \$140 per auctioned tract. The tracts it leaves unoffered are not an opportunity cost but a further saving: because values are correlated, a low value drawn by an early learner means that everyone’s value is likely low, and auctioning such a tract would spend about \$3,965 on learning to realize only about \$1,571 in value, around an eighth of the $\approx$ \$12,756 winning value on a nominated tract.

The largest lever, measured in the separate one-stage exercise, is concentrated attention. The estimated attention function implies that per-tract attention falls in proportion to the number of tracts competing for it. When bidders’ attention is spread across all $\approx$ 28,000 available tracts, few bidders arrive at each tract, so less than 1% of tracts sell. Concentrating the same attention on the offered tracts raises the average arriving bidder count enough that 98% sell—worth about \$8,130 per tract to the seller and \$18,318 to bidders.

Finally, I ask how much the seller would need to know to do without nomination. A seller that observed which tracts are most promising on observable characteristics, and offers tracts arbitrarily only from the top of that ranking, would match nomination’s revenue only by restricting itself to the top 8–10% of the $\approx$ 28,000 available tracts. Even then it would attract fewer bidders (2.7 versus 3.7 per tract) and sell fewer tracts (92% versus 100%), winning only by selling a few of the most promising tracts. Nomination delivers, without any information on the seller’s side, what the seller could otherwise achieve only by knowing

which tracts are the most promising.

1.1 Contributions

This paper provides the first structural analysis of pre-auction nomination: what it is worth, through which channels, and how much a seller would need to know to replicate it without nomination. The closest formal model is [Gorbenko and Malenko \[2024\]](#), who study the initiation of takeover auctions. In their model, two potential buyers who already know their signals can send the seller a costless, non-binding indication of interest, after which the seller decides whether to hold an auction. When values have a large common component, an indication makes the rival revalue the asset upward and bid more aggressively, so bidders prefer not to initiate; when values are mostly private, a bidder with a high enough signal initiates and triggers the auction now rather than risk delay or let the rival initiate. In an extension, they let an uninformed rival pay to learn her signal and show that an initiating bid can preempt that rival. In my setting a nomination works in the opposite direction, encouraging rivals to learn (the information spill over), although the option to wait for someone else’s nomination works against it (under-provision). But like a preemptive bid, the nominator then enters as a committed, guaranteed competitor and competes away the rents of the rivals it draws in.

Because a nomination tells the seller and other bidders that someone is interested in a tract, nomination also connects to work on non-binding signals of interest sent before a sale. In auction design, non-binding indicative bidding serves as a device for screening bidders under costly entry [[Ye, 2007](#), [Quint and Hendricks, 2018](#)], with experimental support in [Kagel et al. \[2008\]](#), though [Ye \[2007\]](#) shows that fully separating equilibria generally fail to exist when the signal is non-binding. M&A sale processes have an analogous non-binding preliminary phase [[Hansen, 2001](#), [Boone and Mulherin, 2007](#)], in which a cheap early signal feeds the later, binding sale stage.

Nomination differs from these signals, and from the indication of interest in [Gorbenko and Malenko \[2024\]](#), in three ways. First, it is costly and binding: the nomination fee is small (\$10 per tract in North Dakota; [Section 2](#)) but nonrefundable, a bidder must learn her value before nominating, and the nominator must buy at the minimum price if no one else bids. This entitles her to win at the reserve when no one else bids, a payoff that non-nominating entrants, who enter only after someone else has triggered the auction, can never obtain. Second, the nominated tract is offered automatically, and potential bidders arrive over time and must pay to learn their values, so a nomination affects whether rivals learn and enter, not only how they bid. Third, values are correlated private values, so what a rival

infers from a nomination is about her own value rather than a shared common value. For less promising tracts, nomination is a costly signal in the sense of [Spence \[1978\]](#): the fee and the learning cost deter nomination, and the purchase commitment makes it unprofitable for a bidder who has learned a low value.

This places nomination closer to the costly-entry tradition of [Levin and Smith \[1994\]](#) and [Bulow and Klemperer \[2009\]](#) than to either non-binding indicative bidding or a competitively bid entry-rights auction: thin early arrival limits how many tracts are nominated, while the learning cost shapes which tracts are nominated. A nomination thickens the market by putting one more committed bidder on every offered tract. [Klemperer and Bulow \[1996\]](#) show that an auction with one more bidder and no reserve raises more revenue than the optimal mechanism with one fewer bidder, and [Coey et al. \[2019\]](#) turn this result into an empirical bound, the bidder exclusion effect.

My decomposition follows the same ranking and the entry-inducement logic of [Bulow and Klemperer \[2009\]](#): almost all of what nomination is worth to the seller comes from the guaranteed bidder rather than from the selection of more valuable tracts. The decomposition by party (Section 7, Table 3) also shows who pays: the seller and the nominator gain, while the rivals that the nomination draws in lose, because they must face a guaranteed competitor. Nomination is also a form of sequential participation. [Roberts and Sweeting \[2013\]](#) show that when entry is costly and selective, letting later buyers condition their entry on earlier entrants' decisions can raise both revenue and efficiency; under nomination, later bidders condition on the fact that someone has nominated.

The findings also connect to the literature on information design in auctions. [Bergemann and Pesendorfer \[2007\]](#), [Board \[2009\]](#), and [Ganuza and Penalva \[2010\]](#) study how much information the *seller* should disclose prior to an auction to maximize revenue. My setting inverts this question: it is *buyers* who choose what information to reveal through nomination, and the seller's instruments (the number of nominations required to trigger an auction, the fee, and the purchase commitment) determine how costly demand revelation becomes. Because buyers bear the cost of generating this information, a nomination is a public good among bidders: each would rather another firm pay to learn and nominate and then learn later, an incentive that does not arise in standard seller-disclosure frameworks.

A motive to delay learning in order to observe rivals also appears at the exploration stage of this industry: [Hodgson \[2024\]](#) estimates a dynamic model of UK offshore drilling in which firms delay costly exploratory drilling to observe rivals' well results. There, the information spilling over is a signal about a commonly valued unknown (the location of oil); here, what a bidder infers from a rival's nomination is about her own private value, transmitted through the positive dependence of values.

The concentrated-attention channel connects nomination to work on how limited attention is spread across many alternatives.⁵ In the sparse-max model of [Gabaix \[2014\]](#), a decision maker attends only to the parts of a problem that matter enough relative to the cost of attention; read as a model of which tracts a firm arrives at, it predicts that most tracts receive no attention at all. When the choice set is large, decision makers seriously consider only a small subset of it, and an option’s chance of entering that subset depends on whether something brings it to their notice, not only on its value [[Goeree, 2008](#), [Abaluck and Adams-Prassl, 2021](#), [Crawford et al., 2021](#)].

A nomination plays that role here: it places the tract in the sale notice that every potential bidder reads. In rational-inattention models, alternatives with higher prior expected value are more likely to be considered [[Caplin et al., 2019](#)]; consistent with this, the share of available tracts that are nominated falls by about two orders of magnitude from the most to the least promising tracts (Appendix C.1). The same dilution appears in financial markets, where the immediate response to a firm’s earnings announcement is weaker on days when more other firms announce [[Hirshleifer et al., 2009](#)], and on eBay, where [Coey et al. \[2020\]](#) estimate that buyers attend to only about 6% of the relevant auctions they encounter and that auctions become unviable when few buyers pay attention. [Bodoh-Creed et al. \[2021\]](#) show that consolidating trade into fewer auctions removes much of the allocative inefficiency of a platform with many simultaneous auctions, though at some cost to seller revenue. Nomination achieves a similar consolidation through buyers rather than the platform, shrinking the offered set from about 28,000 available tracts to a few dozen per season.

Finally, I extend affiliated private value (APV) empirical methodologies, developed for settings where all bids are observed [[Li et al., 2002](#), [Athey and Haile, 2002](#)], to a setting with both a censored highest bid and positively dependent values. When the full bid profile is not observed, values are not nonparametrically identified under standard affiliation assumptions [e.g., [Athey and Haile, 2002](#)]. Under independence, [Espín-Sánchez et al. \[2025\]](#) show that the number of bidders and the marginal value distribution are nonparametrically identified from any two order statistics, so the highest bid need not be observed; their framework identifies dependence only when the copula is max-stable (e.g., CES or Gumbel-type extreme-value copulas), and only from the two highest order statistics. In my setting the estimated positive dependence need not be max-stable and the highest order statistic is censored, and no general nonparametric identification result covers both at once. I address this gap by pairing a parametric copula for the dependence structure with a flexible approximation to the marginal.

⁵For surveys, see [Gabaix \[2019\]](#) and [Loewenstein and Wojtowicz \[2025\]](#).

2 Institutional Details

The North Dakota (ND) state government holds quarterly auctions for oil and gas lease sales through the Department of Trust Lands. Each cycle, referred to as a *season* throughout this paper, begins almost immediately after the previous auction concludes. At this time, the state announces the date of the next auction and calls for nominations from interested parties along with the deadline for nominations. The list of tracts open for leasing is posted on its website. Tracts need at least one nomination to be considered for auction. The size of the tract can vary, ranging from 0.01 acres to a maximum of 160 acres as specified by state regulations. The nomination process requires a nonrefundable \$10 fee per tract. Nominators can determine the minimum opening bid. If not specified, the opening bid is \$2 per acre. I treat this minimum opening bid as the auction’s reserve price.

Following the nomination period, the state conducts a sequence of ascending price auctions to individually sell nominated tracts. Each tract opens for bidding individually, with auctions closing after a 7-day period, sequentially at two-minute intervals. Closing times are fixed and never extended. Bids are received at per-acre level. If no one bids, the nominator becomes the automatic winner and has to pay the minimum opening bid they specified.

Upon winning a lease auction, the successful bidder pays a one-time bonus (the per-acre bid times the acreage), a one-time ND administrative fee of \$10, a five-year prepaid rental of \$1 per net acre per year, and a 2% buyer premium on the bonus.⁶ Once a well is drilled and begins producing, the leaseholder pays a royalty rate, which is the percentage of oil or gas revenue paid to the mineral rights owner, of either 3/16ths or 1/6th, depending on lease terms.⁷

The winning bidders obtain exclusive rights to explore, drill, and produce oil and gas from their tracts for a maximum lease period of five years. Extensions beyond this period are only permitted if the tract is being actively developed and producing.

Bidders have access to what is called *proxy bidding*, an online system that bids on behalf of the user until the highest competing bid exceeds the value the user has set as their *proxy bid*. In other words, as long as the competing bidders are bidding lower than the proxy bid value, the proxy bidder will keep its winning status with a bid incrementally higher (\$1) than the highest competing bid. If the competing bid is equal to the proxy bid, say at \$10, but was placed after the proxy bid was placed, the proxy bidder will show up as the winner at \$10 and the bidder who bid \$10 but placed after the proxy bidder will lose with their bid

⁶For example, a \$2 per-acre bid on 40 net acres costs \$80 in bonus, \$10 in fees, \$200 in rental, and \$1.60 in buyer premium, a total initial cost of \$291.60.

⁷Royalty rate differs by county, and most counties are one/sixth (1/6) except for counties like Billings, Divide, Dunn, Golden Valley, McKenzie, Mountrail and Williams.

recorded as \$9.

Under a proxy bidding system, if values are private (as I assume in Section 4), the optimal bidding strategy of the bidders is to place a proxy bid at their private value. For this reason, most winning bids are a dollar higher than the second highest bid. In this sense, the winning bid is *censored*. Sometimes bidders will not use this proxy bid system. When this happens, a distinct *jump* is observed in the data. For example, instead of placing a proxy bid of \$10 after the previous bidder placed a proxy bid of \$5, the next highest recorded bid will be \$10 instead of a bid incrementally higher than \$5, which in this case is \$6. Such instances are rare and informative in that they reveal the winner’s true bid directly. However, I do not explicitly use these jump-bid auctions and they enter the estimation on equal footing with all other auctions. In addition, I abstract away from the fact that the losing bidder’s bid may be lower than what is recorded due to this system.

Bidding is concentrated towards the end of each auction (Appendix C.9). Half of all bidders place their first bid within the last ten minutes before the close, and the decisive bids come later still: the winning bidder’s last bid arrives a median of 0.7 minutes before the close and the runner-up’s 0.6 minutes before, with 77% and 85% of them, respectively, in the final 30 minutes. A smaller group bids early: 29% of bidders first bid more than six hours before the close.⁸ Few of the last-minute bids look like a proxy bid answering a rival: of the 1,410 bids placed in the final minute, 141 (10%) arrive within five seconds of a rival’s bid and exceed it (Table 22). I take each bidder’s last bid as her bid.

Revisions are uncommon: 71% of bidder–tract pairs record a single bid, and of the 29% that revise, about half revise only once; ten or more revisions are rare (1%). Revision is also less common among bidders who enter closer to the end: 41% of those who first bid more than a day before the close revise, against 27% of those who first bid 5–30 minutes before it and 11% of those who first bid in the final minute (Figure 21). I interpret the bidders who revise as less sophisticated bidders, unfamiliar with the proxy system, who raise their bid in steps, so that their last bid is their true bid. That repeated revision is rare is also hard to reconcile with bidders learning about their values during the auction, which would lead them to update their bids repeatedly.⁹ Two caveats apply. The observed revisions are a lower bound on the bidders who would have liked to revise, because a bidder cannot usefully raise her bid once the standing price has passed her value. And bidders who enter close to

⁸Late bidding is also the norm on eBay, where the median winning bid arrives 1.4 minutes before the close [Hickman et al., 2017]. Bajari and Hortaçsu [2003] model the final stage of eBay auctions as a sealed-bid auction.

⁹I assume that bidders’ values are private and a bidder learns her value before she bids. What a bidder sees of her rivals during the auction does not change her own value, and bidding that value remains optimal whatever she comes to believe about her rivals.

the end have little time to revise, which is consistent with the lower revision rate among late entrants. Because the decisive bids arrive in the final minutes and most bidders bid once, each bidder’s final proxy bid acts as a single sealed bid.

Part of what makes a tract valuable is public: tracts close to existing wells and larger tracts draw higher bids. Throughout the paper I refer to tracts that are far from existing wells or small in size as less promising, and to tracts that are close to existing, recently developed wells or large as more promising (Section 5.1). While the broad geological formation underlying a given region is public knowledge, translating that shared information into a firm-specific dollar value for a particular tract is not costless. For instance, identifying whether a given tract sits within such a reservoir – a localized pocket within the broader formation where the rock is naturally fractured or under sufficiently high pressure to make extraction economical – and how much it would cost a specific firm to develop it requires firm-specific analysis. Phone interviews with market participants conducted for this project confirm that this evaluation step is a real and heterogeneous cost across firms: larger firms typically maintain in-house land men and geologists (or even their “bosses” with years of experience are capable of such evaluation on the spot from “looking at the map”) to evaluate prospective tracts, while smaller firms rely on third-party consultants whose availability, and hence cost, is uncertain from one tract to the next. This heterogeneity in the cost of acquiring firm-specific information motivates modeling this cost, which I refer to as the *learning cost*, as privately observed and drawn independently across bidders, rather than as a fixed or publicly known cost of participation.

3 Data

The main data for estimation are auction-level bid data and nomination information for each auction in 2017–2018. They combine (1) auction-level bid data for all bidders, scraped from the online platform that runs the auctions, and (2) nomination data. Since the National Defense Authorization Act (NDAA) for Fiscal Year 2015 amended the Mineral Leasing Act to allow internet-based lease sales, many state mineral-rights auctions have run on the bidding platform EnergyNet, whose bidding histories are public. Additional information about each auction, such as royalty rate, size in acres of the tract, identity of nominator, and start and end times of the auction publicized during the auction, is available as well. Detailed information on nominations received per tract was acquired through a Freedom of Information Act (FOIA) request. Bidder identity during auction is anonymized using bidder id numbers. The winning bidder’s identity is revealed. Using the bidder id and winning bidders’ names, bidders who have won at least once during the given season can be traced within the season.

Bidders who never won stay anonymous. This means firm-level characteristics (size, drilling technology, or existing lease portfolio) are observed for only a small subset of bidder-tract pairs (winners within a season). Consequently, I do not model them explicitly as bidder-level heterogeneity.

Supplementary data are needed to understand which lands were available for nomination for each auction. This information is reconstructed using (1) the comprehensive list of tracts available for leasing and (2) the auction history for 2012–2016. Potential nominators can check current lease status and available tracts for nomination through the public records of the ND Department of Trust Lands. These records include detailed geographical information according to the World Geodetic System 1984 (WGS 84) coordinate system for tracts that are currently leased and tracts available for lease. They also include information on the most recent past lease associated with each tract and whether the lease was extended beyond its maximum five-year lease period.¹⁰ Past auction history for both offline (before the NDAA was passed) and online auctions is acquired from public data of the ND Department of Trust Lands. Combining these two data sources, I reconstruct the list of tracts that would have been available for nomination by the time a potential nominator arrived to nominate. Figure 7 in Appendix A maps the available tracts and the nearest producing wells.

There are 878 auctions in total, covering 2,740 bids. Below, I report the summary statistics from the entire data unless specified otherwise. The number of auctions used for estimation is 728 covering 1,875 bids. I keep only the auctions where there is more than one competitor and auctions where the size is greater than 0.01 acres.

Table 1 breaks these counts down by season. The second row reports the number of auctions triggered, i.e. the tracts that received at least one nomination. The ‘Tracts Available’ column reports the number of tracts that would have been open for nomination, constructed from the tract list and the auction history of the previous five years. The table also reports the number of parcels that received two and three nominations, obtained through a FOIA request, and the total number of bids received in each season. During 2017–2018, a total of 878 tracts were nominated and auctioned, which is about 110 per season, or roughly 0.4% against the approximately 28,000 tracts available in a given season ($\approx 110/28,000$).

Figure 1 compares the number of bidders and nominators per tract. While the number of bidders per auction ranges from one to 11 with a median of three, at most three bidders nominate a tract, and the majority of auctions are triggered by a single nominator, even though nominating costs only \$10. Appendix A reports the full distribution of bidders per auction (Figure 8) and the number of unique bidders active in each season (Table 13).

¹⁰To the best of my knowledge, based on the most current lease data, most leases terminate after the five-year period unless a new well was drilled.

Table 1: Summary of Available Tracts, Nomination Counts and Bids Received

Season	# Auctions	Tracts Available	Nom. $\times 2$	Nom. $\times 3$	# Bids Received
2017-Q1	173	27,867	21	2	361
2017-Q2	234	27,885	0	0	661
2017-Q3	166	27,885	0	0	504
2017-Q4	67	27,965	0	0	270
2018-Q1	52	28,062	2	0	240
2018-Q2	58	28,064	0	0	191
2018-Q3	29	28,408	0	0	148
2018-Q4	99	28,461	0	0	365
Total	878		23	2	2,740

Notes: Table summarizes the number of auctions and nominations by season during 2017–2018. The ‘Tracts Available’ column shows the total number of tracts that could have been nominated. The ‘Nom. $\times 2$ ’ and ‘Nom. $\times 3$ ’ columns indicate the number of tracts that received multiple nominations.

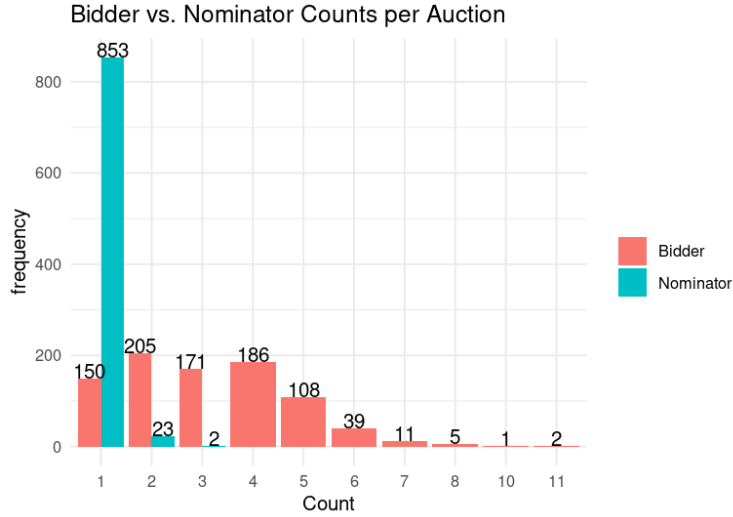


Figure 1: Histogram of per Auction Bidder vs. Nominator

Notes: Histogram of the number of bidders and the number of nominators per auction.

Figure 2 shows the same pattern at the bidder level, for the bidders whose participation can be traced within a season because they won at least once. Each point is a nominator-auction pair: the horizontal axis counts all tracts the bidder participated in during that auction, and the vertical axis counts the tracts they nominated; points on the 45-degree dashed line are bidders who nominated every tract they participated in. Most observations (green circles) lie along the horizontal axis, with almost no nominations despite participation in dozens or even over a hundred tracts. Only a handful of nominators (orange triangles) lie near the 45-degree line, and even they learn later on a substantial share of the tracts they enter. Together, the two figures show that learning later is pervasive.

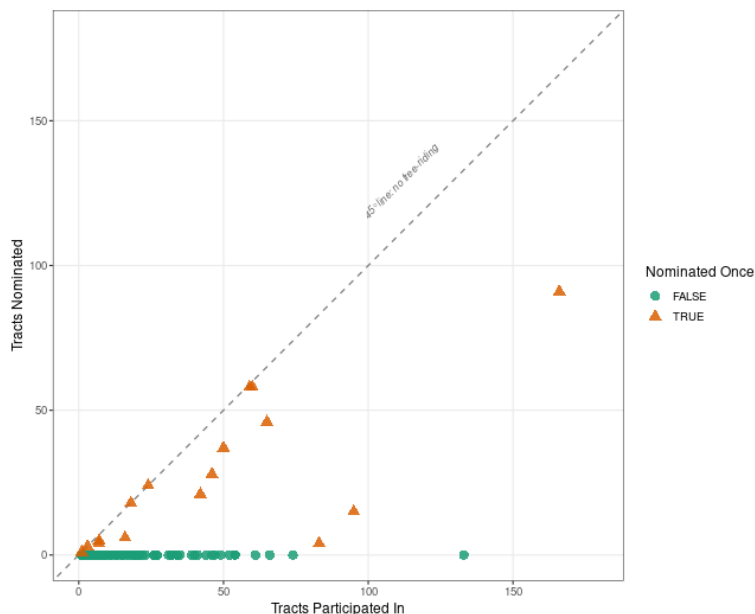


Figure 2: Tracts Participated vs. Tracts Nominated, by Nominator-Auction Pair

Notes: Each point is a bidder-auction pair restricted to bidders whose identities are known. The horizontal axis is the total number of tracts the bidder participated in (bid on); the vertical axis is the number of tracts they nominated.

Phone interviews with market participants suggest that not nominating is a deliberate strategic choice. One non-nominating bidder reasoned that it is “unlikely there’s anything valuable if no one else nominated it” and described nominating as “a waste of time” when they can instead “rely on other nominators” to trigger the auction on their behalf. The model captures this choice: nominating requires paying the learning cost and the \$10 fee up front, while learning later defers the learning cost until the auction is confirmed to occur.

I do not model tract quality explicitly, as not all tracts have realized oil and gas production data available. Instead, I use bids as a revealed-preference measure of tract value: each bid reflects the bidder’s perceived value of holding the exclusive right to explore, drill, and extract

from that tract over the five-year lease period, incorporating their private assessment of geological prospects, extraction costs, and future commodity prices. The bid homogenization procedure in Appendix B.1 controls for observable tract characteristics, including geographic location, isolating the private value component.

Summary statistics on bid values, parcel sizes, royalty rates, nearest-well characteristics, and starting bids are reported in Appendix A (Tables 5–12).

4 Model

I model nomination and auction as two separate stages. Potential bidders, or “buyers”, arrive sequentially and must incur a cost to learn their private values. They face strategic decisions about when to learn their values and nominate. The auction only proceeds if sufficient nominations are received, creating coordination and late-learning incentives.

4.1 Game Setup

I model the sale of a single tract t to a population of potential bidders who arrive over time; the total number that ever arrive, I_t , is random and determined endogenously by the Poisson arrival processes described below. Bidders are symmetric, and conditional on I_t bidders arriving, their private values $V_t = (V_{1t}, \dots, V_{I_t t})$ are exchangeable and jointly distributed according to H_V on a common support $[0, \infty)^{I_t}$.¹¹ The marginal CDF $F_V(v_{it})$ is then defined as $H_V(\infty, \dots, v_i, \dots, \infty)$.

I allow for positive dependence across values so that for V_{it}, V_{jt} ($i \neq j$), $F_V(v_{jt} \mid v'_{it}) \leq F_V(v_{jt} \mid v_{it})$ for all $v'_{it} > v_{it}$. This captures the reality that firms observing similar information (production from nearby wells or market conditions) form positively correlated value estimates.¹²

This modeling choice is consistent with a broader shift in the recent empirical auction literature away from common-value frameworks for onshore oil and gas lease sales [Covert and Sweeney, 2023].¹³ Horizontal drilling and hydraulic fracturing target a broad source rock rather than a narrow, hard-to-locate deposit, reducing the production uncertainty that historically justified a common-value treatment of these auctions [Kong, 2020]. Differences in bids are therefore driven primarily by bidder-specific factors (development strategy, drilling

¹¹ H_V denotes this consistent family of exchangeable distributions, indexed implicitly by the realized number of arrivals.

¹²This is the affiliated private values (APV) property of Milgrom and Weber [1982]; see Appendix D.1 for the formal definition and for verification that the copula adopted in Section 5 satisfies it.

¹³See, e.g., Hendricks and Porter [1988] for the classic common-value treatment of offshore auctions.

technology, portfolio fit, and cost of capital). Zero correlation across bidders would nonetheless be too strong: bidders drawing on the same regional geological and market information are likely to reach correlated, if not identical, conclusions about a tract’s value.¹⁴ Correlated private values take the middle path: values are private and costly to learn, but positively correlated through a common factor reflecting shared uncertainty about the region’s geology and oil and gas prices.

No bidder observes any realization of V_t ex ante, including her own, and must incur a learning cost ($c_{it} \sim F_C$, i.i.d.) to learn her value, i.e. paying c_{it} reveals v_{it} to buyer i alone. This learning cost is privately observed and independent of V_t .

There are three policy instruments exogenously given and common across tracts: 1. the nomination requirement, or the number of nominations a tract must receive to be offered for sale, $K \in \{0\} \cup \mathbb{N}$; 2. the per-nomination fee $\phi \geq 0$; and 3. the reserve price $r \geq 0$. These are public. In North Dakota, $K = 1$ and $\phi = \$10$.

I describe the model below for a single tract, suppressing tract-level heterogeneity; Section 4.5 extends the model to heterogeneous tracts and introduces the observable tract characteristics X_t .

The sale proceeds in two stages: the nomination (Stage 1) and the auction (Stage 2). In Stage 1, the seller receives nominations for t . If total received nominations for this tract (M_t) reaches the requirement ($M_t \geq K$), t is sold in a second-price sealed-bid auction in Stage 2.¹⁵ The realized nomination count M_t is not revealed. Bidders arrive according to two independent, homogeneous Poisson processes, one for each stage.¹⁶ I use λ_{pre} and λ_{post} to represent the arrival rates of Stage 1 and Stage 2, respectively. I normalize each stage to one unit of time, so each arrival rate is the expected number of bidders who arrive at a tract in that stage, or the stage’s *average arriving bidder count*.¹⁷

An important timing distinction clarifies how positive dependence in H_V operates. A bidder who has not yet paid c_{it} to learn can update her beliefs about her own yet-to-be-

¹⁴Krasnokutskaya and Seim [2011] make a related point in the context of highway procurement auctions, arguing that correlation in bids can arise either from heterogeneity that is observable to bidders but not to the econometrician, or from genuine correlation of private costs, and that distinguishing between the two is generally not possible without strong auxiliary assumptions.

¹⁵The auction is run as an ascending auction with proxy bidding, but the decisive bids arrive in the final minutes and most bidders bid only once, so each bidder’s final proxy bid acts as a single sealed bid (Section 2; Appendix C.9). Following Bajari and Hortaçsu [2003], I treat it as a second-price sealed-bid auction. Because values are private and learned before bidding (Assumption A1), nothing a bidder observes during the auction changes her value, so bidding her value remains optimal.

¹⁶Nomination counts in my data are overdispersed relative to a Poisson (Appendix B.7), as are bidder arrivals at eBay auctions [Hickman et al., 2017]; I keep the standard Poisson for tractability.

¹⁷I normalize because I do not observe how long each stage lasts. Normalization lets arrival times within a stage play no role in the model, so only the expected number of arrivals per stage matters. A stage of length T with arrival rate λ/T gives the same equilibrium.

learned value upon observing nomination signals from others. Let $n(v) \in \{0, 1\}$ denote a bidder's (as yet unspecified) nomination rule, mapping a realized value v to a nomination decision, and let $\mathcal{E}_m \equiv \{n(V_1) = 1, \dots, n(V_m) = 1\}$ be the event that exactly m other bidders have nominated. By Bayes' rule and the joint distribution H_V , being exchangeable and symmetric across bidders, the posterior density of bidder i 's own value conditional on $\mathcal{E}_m$ is

$$f_{V_i|\mathcal{E}_m}(v) = \frac{\int_{\mathbb{R}_+^m} h_{m+1}(v, u_1, \dots, u_m) \prod_{j=1}^m \mathbb{I}\{n(u_j) = 1\} du_1 \cdots du_m}{\Pr(\mathcal{E}_m)}, \quad (1)$$

where h_{m+1} is the $(m+1)$ -dimensional marginal density of H_V and $\Pr(\mathcal{E}_m) = \mathbb{E}\left[\prod_{j=1}^m \mathbb{I}\{n(V_j) = 1\}\right]$ is the normalizing constant. Section 4.4 shows that the equilibrium nomination rule takes the threshold form $n(v) = \mathbb{I}\{v \geq v^*\}$, in which case $\mathcal{E}_m$ reduces to $\{V_1 \geq v^*, \dots, V_m \geq v^*\}$ and the integral above is taken over $(v^*, \infty)^m$; this is the form used throughout the rest of the paper. Under positive dependence of H_V and any weakly increasing nomination rule $n(\cdot)$, the posterior stochastically dominates the prior:

$$F_{V_i|\mathcal{E}_m}(v) \leq F_V(v) \quad \text{for all } v, \quad (2)$$

because knowing that others nominated raises the probability that V_i is also high.¹⁸¹⁹

4.2 Strategy Set

I assume the following:

Assumption A1. *Learning is required before nominating or bidding.*

Under Assumption A1, I restrict the strategy set of buyers accordingly: l_i , or learning timing, for bidders who arrived in Stage 1; n_i , or nomination decision for bidders who learned their values; d_i , or learning decision for bidders who arrived in Stage 2.

Potential bidder i arriving during Stage 1 chooses a learning timing $l_i \in \{1, 2, 3\}$. **Stage 1 (early) learner** ($l_i = 1$) pays learning cost c_{it} immediately and learns private value $v_{it} \sim F_V(\cdot)$. Nomination decision $n_i = n(v_{it}) \in \{0 \text{ (don't nominate)}, 1 \text{ (nominate)}\}$ is made based on v_{it} .

¹⁸This follows directly from positive correlation of H_V and holds for any copula satisfying that property, not only the Gaussian copula adopted in estimation; see Appendix D.1.

¹⁹A simplified version of this game—stripping out value uncertainty and costly learning—is solved in Appendix E.1 to build intuition for how the nomination requirement K shapes entry and the probability that a tract receives at least K nominations.

Stage 2 (late) learner ($l_i = 2$) waits without learning v_{it} and pays the learning cost c_{it} to learn in Stage 2 only if the auction proceeds. **Exit** ($l_i = 3$) means leaving without learning their private value. Let $\pi_{l=1}$ and $\pi_{l=2}$ be the ex-ante probability of a randomly arriving bidder choosing $l = 1$ and $l = 2$ respectively.

If Stage 2 (the auction stage) is triggered ($M_t \geq K$), more potential bidders arrive according to $\text{Poisson}(\lambda_{\text{post}})$. Upon observing c_{it} , they decide whether to enter ($d_i \equiv d(c_{it}) \in \{1 \text{ (Learn)}, 0 \text{ (Exit)}\}$). $d_i = 1$ bidders observe their values. Let $\pi_{d=1}$ be the ex-ante probability of a randomly arriving bidder in Stage 2 choosing $d = 1$. Bidders who chose $l_i = 2$ in Stage 1 also learn their value by paying the learning cost c_{it} . All potential bidders who have learned their value ($l \in \{1, 2\}$ from Stage 1 and $d = 1$ from Stage 2) enter the auction and bid $b_i(v_i)$ if the value exceeds reserve price ($v_{it} \geq r$) and exit otherwise (Figure 3).

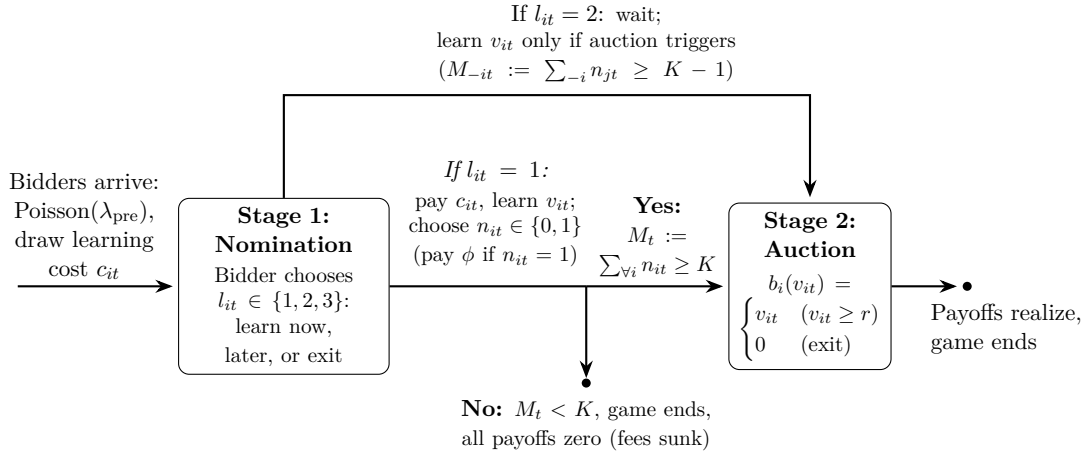


Figure 3: Timing of the model.

4.3 Payoffs

4.3.1 Stage 2: Expected Auction Surplus

A bidder with realized value v who enters the auction bids $b(v) = v$ (weakly dominant strategy under second-price rules) and wins if and only if her value exceeds both the reserve r and the highest competing value. Given the joint value distribution H_V , the expected surplus given j total competitors is

$$\tilde{\Pi}(v; j, H_V) = \int_r^v (v - y) f_{Y|v}(y | v, j, H_V) dy, \quad (3)$$

where $Y \equiv \max\{V_1, \dots, V_j\}$ is the highest competing value and $f_{Y|v}$ is its conditional density given own value v , accounting for positive dependence through H_V (derived in Ap-

pendix B.4).²⁰ With $j = 0$, $\tilde{\Pi}(v; 0, H_V) = v - r$ when $v \geq r$ and zero otherwise.

When a bidder faces m *deterministic* competitors (realized nominators) and a Poisson(Λ) number of *stochastic* competitors (non-nominating early arrivals and late arrivals), the expected surplus integrates over the Poisson distribution:

$$\Pi(v; m, \Lambda, H_V) = \sum_{n=0}^{\infty} \frac{e^{-\Lambda} \Lambda^n}{n!} \tilde{\Pi}(v; m + n, H_V). \quad (4)$$

In this setup,²¹ the total expected stochastic competition is the sum of non-nominating bidders with values above the reserve coming from three different groups: (1) later-learners from Stage 1 ($l = 2$); (2) early learners from Stage 1 ($l = 1$) who did not nominate but participate in the auction; and (3) Stage 2 arriving learners ($d = 1$).

4.3.2 Stage 1: Learning and Nomination

Bidders arriving at Stage 1 choose when to learn their values. The payoffs of the three strategies ($l_i \in \{1, 2, 3\}$) differ in when the learning cost c_i is paid, in how many other nominations are needed to satisfy $M \geq K$, and, under positive dependence, in beliefs about the yet-to-be-learned value. Only bidders who choose $l_i = 1$ can nominate ($n_i = 1$), paying the fee ϕ (Assumption A1).

$l_i = 1$ (Learning Early) Payoff. For a bidder with learning cost c_i , the value of learning immediately is

$$\begin{aligned} U_{l_i=1}(c_i) = & -c_i + \int_{\underline{v}}^{\bar{v}} \mathbb{I}\{n(v) = 1\} \cdot \left[-\phi + \sum_{m=K-1}^{\infty} \Pr(M_{-i}=m) \Pi(v; m, \Lambda_{nn}, H_V) \right] f_V(v) dv \\ & + \int_{\underline{v}}^{\bar{v}} \mathbb{I}\{n(v) = 0\} \cdot \sum_{m=K}^{\infty} \Pr(M_{-i}=m) \Pi(v; m, \Lambda_{nn}, H_V) f_V(v) dv, \end{aligned} \quad (5)$$

where $-c_i$ is the learning cost paid immediately upon choosing $l_i = 1$, $[\underline{v}, \bar{v}]$ is the support of F_V , $\Pr(M_{-i} = m)$ is the probability that the number of other nominators (M_{-i}) is m , and Λ_{nn} is the expected number of non-nominating competitors who bid; Section 4.4 derives both from the equilibrium strategies (equations (25)–(26)). Having learned, a bidder enters and bids whenever $v \geq r$, so her continuation value has two branches. The first (bracketed)

²⁰Note that every competitor has $v \geq r$, so $Y \geq r$ because bidders can enter only if they clear the reserve.

²¹Appendix D.9 describes how the Poisson sum in equation (4) is truncated in computation.

term applies when she nominates ($n(v) = 1$, i.e. $v \geq v^*$):

$$-\phi + \sum_{m=K-1}^{\infty} \Pr(M_{-i}=m) \Pi(v; m, \Lambda_{nn}, H_V) \quad (6)$$

is the payoff from nominating given v : she pays the fee ϕ and earns the expected auction surplus for each realized number $M_{-i} = m$ of other bidders' nominations; the sum starts at $m = K - 1$ because her own nomination brings the total to $m + 1 \geq K$. The second term applies when she has learned but does not nominate ($n(v) = 0$): she pays no fee and collects auction surplus only when others' nominations reach the threshold ($m \geq K$).

$l_i = 2$ (Learning Later) Payoff. A buyer with cost c_i who chooses $l_i = 2$ pays nothing in Stage 1. If the auction triggers ($M \geq K$), she updates her beliefs about her own value using the posterior implied by H_V upon observing the auction was triggered ($\mathbb{I}\{M \geq K\}$, but not M itself), pays c_i to learn v_i , then enters if $v_i \geq r$. The expected payoff is

$$U_{l_i=2}(c_i) = \sum_{m=K}^{\infty} \Pr(M=m) \left[-c_i + \int_r^{\bar{v}} \Pi(v; m, \Lambda_{nn}, H_V) f_V(v | \mathcal{E}_m) dv \right], \quad (7)$$

where M counts only other bidders' nominations. The competition argument is Λ_{nn} because conditioning on your own arrival does not change the distribution of the other arrivals.²² $f_V(v | \mathcal{E}_m)$ is the posterior over her own value after observing $\mathcal{E}_m$, the event that m other bidders nominated (equation (1)), derived from H_V . By the stochastic dominance result in equation (2), this posterior shifts the non-nominating bidder's value distribution weakly upward relative to the unconditional $f_V(v)$.

$l_i = 3$ (Exit) Payoff. Bidders who choose to exit never return to the game and earn zero payoff.

$d_i = 1$ (Late Arrivals' Learning) Payoff. A Stage-2 post-arrival who observes the auction triggered ($M \geq K$) faces a similar decision as the $l_i = 2$ strategy: pay learning cost c , learn v , and bid if $v \geq r$. The two problems differ only in when the learning cost is sunk.

Conditional on exactly m nominations, a post-arrival and an $l = 2$ later learner update their value beliefs to the same posterior $f_V(v | \mathcal{E}_m)$ (from H_V) and face the same Poisson pool of other non-nominator rivals. They differ only in the timing of paying the learning cost: a post-arrival exists only in a triggered auction and therefore pays c_i *for sure*, whereas

²²This follows from the environmental equivalence property of Poisson games. See Myerson [1998].

an $l = 2$ bidder pays c_i only if the auction triggers. The post-arrival's expected payoff is thus

$$U_{\text{post}}(c_i) = -c_i + \sum_{m=K}^{\infty} \Pr(M=m \mid M \geq K) \int_r^{\bar{v}} \Pi(v; m, \Lambda_{\text{nn}}, H_V) f_V(v \mid \mathcal{E}_m) dv. \quad (8)$$

4.4 Equilibrium

I characterize a symmetric Perfect Bayesian Equilibrium (PBE) in which all arriving Stage 1 bidders follow the same cost-cutoff strategy $l_i = l(c_{it}) \in \{1, 2, 3\}$.²³

Bidders hold rational expectations about the distribution of competitors, which depends on the equilibrium probabilities $(\pi_{l=1}, \pi_{l=2})$; an equilibrium is a fixed point in these probabilities (Definition 2).

I solve the model by backward induction. In **Stage 2**, truthful bidding, $b(v_{it}) = v_{it} \cdot \mathbb{I}\{v_{it} \geq r\}$, is weakly dominant (Section 4.3.1). In **Stage 1**, bidders follow threshold strategies: an early learner nominates when her value exceeds v^* ; Stage-1 arrivals with $c < c_L^*$ choose $l = 1$ and those with $c_L^* \leq c \leq c_H^*$ choose $l = 2$; and late arrivals learn ($d = 1$) when $c < c_{\text{post}}^*$.

The learning payoffs in Section 4.3.2 are linear and decreasing in the learning cost c , at different rates for $l = 1$ and $l = 2$ (Proposition 1), so the learning decision is a cutoff rule in c . The nominator's expected auction surplus, the left-hand side of (10) below, is not linear in v , and with positively correlated values its monotonicity is not automatic: a higher own value raises the surplus against any given rivals, but it also signals that rivals' values are higher. At the estimates the surplus is strictly increasing in v (Figure 14, Appendix C.4), so the nomination decision is also a cutoff rule in v . Therefore, for each realized (and learned) v and c , cutoff strategies are determined using indifference conditions.

Nomination Threshold v^* . The nomination threshold is

$$v^* = \max\{r, v^{**}\}, \quad (9)$$

where v^{**} satisfies

$$\sum_{m=K-1}^{\infty} \Pr(M_{-i} = m) \Pi(v^{**}; m, \Lambda_{\text{nn}}, H_V) = \phi. \quad (10)$$

Intuitively, v^{**} is the value at which a bidder is exactly indifferent between nominating and not nominating given expected competition and the per-nomination fee ϕ . When $v < v^{**}$, the

²³With incomplete information over private costs and values, the appropriate solution concept is a symmetric perfect Bayesian equilibrium (PBE). Because Stage 2 has a unique Nash equilibrium (truthful bidding is weakly dominant), backward induction uniquely pins down Stage 2 behavior, and Stage 1 strategies are best responses given correct beliefs. The resulting equilibrium is therefore PBE.

fee exceeds the expected surplus from participation. The reserve r imposes an institutional floor: any bidder with $v < r$ cannot profitably participate even unopposed.

This specializes the general nomination-event definition in equation (1) to the threshold form $\mathcal{E}_m = \{V_1 \geq v^*, \dots, V_m \geq v^*\}$, with $n(v) = \mathbb{I}\{v \geq v^*\}$. This form is used in the payoffs for $l_i = 2$ and $d_i = 1$ calculations.²⁴

The purchase commitment gives a nominator a payoff that a later learner never receives when $K = 1$: a lone nominator wins at r and earns $v - r \geq 0$, whereas later learners enter only once $M \geq K \geq 1$ and so can never be the sole bidder at auction.²⁵ The cost of learning early is that the learning cost c_i is sunk even when the bidder draws $v < v^*$ and does not nominate. Nomination is discouraged only by the learning cost c_i and the fee ϕ , both of which enter the payoff unconditionally; the purchase commitment does not.

Plugging v^* into (5), since $\mathbb{I}\{n(v) = 1\} = 1$ when $v \geq v^*$ and zero otherwise, (5) can be rewritten as $-c_i + \int_{v^*}^{\bar{v}} U_{n_i=1}(v) f_V(v) dv$, where $U_{n_i=1}(v)$ is the nomination payoff (6). Since the nomination fee is paid with probability $1 - F_V(v^*)$,

$$U_{l_i=1}(c_i) = -c_i - \phi \cdot [1 - F_V(v^*)] + \int_{v^*}^{\bar{v}} \sum_{m=K-1}^{\infty} [\Pr(M_{-i}=m) \Pi(v; m, \Lambda_{nn}, H_V)] f_V(v) dv \quad (11)$$

$l = 1, l = 2$ Thresholds c_L^* and c_H^* . The cost cutoffs follow from comparing $U_{l=1}(c_i)$ and $U_{l=2}(c_i)$.

Proposition 1 (Single Crossing). *The payoff functions cross at most once and there exists at most one c_L^* such that $U_{l=1}(c_L^*) = U_{l=2}(c_L^*)$.*

Proof. See Appendix D.7. □

Let $A = \int_{v^*}^{\bar{v}} \sum_{m=K-1}^{\infty} \Pr(M_{-i}=m) \Pi(v; m, \Lambda_{nn}, H_V) f_V(v) dv$ from (11) be the expected gross auction surplus for a $l = 1$ player and

$B = \sum_{m=K}^{\infty} \left[\Pr(M=m) \int_r^{\bar{v}} \Pi(v; m, \Lambda_{nn}, H_V) f_V(v | \mathcal{E}_m) dv \right]$ from (7) be the expected gross auction surplus for $l = 2$ player.

Both payoffs are linear in c_i : $U_{l=1}$ falls one-for-one, while $U_{l=2}$ falls only at rate $\Pr(M \geq K) < 1$, because a later learner pays the learning cost only if the auction triggers. Depending on the intercepts and slopes, exactly one of three equilibrium cases holds.

²⁴The pure-strategy threshold rule is robust to noisy/mixed nomination strategies. See Appendix D.2 for the formal argument.

²⁵The $j = 0$ term inside $\Pi(v; K-1, \Lambda_{nn}, H_V)$ captures exactly this: a strictly positive contribution to nominators' expected surplus that is structurally unavailable to non-nominators.

Proposition 2 (Equilibrium Structure). *Define*

$$\Delta_0 := U_{l=1}(0) - U_{l=2}(0) = A - \phi[1 - F_V(v^*)] - B \quad (\text{payoff difference at zero cost}) \quad (12)$$

$$\Delta_{l=2} := U_{l=1}(c_H^*) - U_{l=2}(c_H^*) = U_{l=1}(c_H^*) \quad (\text{payoff from choosing } l = 1 \text{ at } c_H^*) \quad (13)$$

Then exactly one of the following holds. The empirically relevant case is Case 1 (interior solution), described below. The two corner solutions in Case 2 (coordination failure: everyone waits and learns later) and Case 3 (everyone learns early) are presented in Appendix D.4.

Case 1: Interior Solution (Single Crossing) If $\Delta_0 \geq 0$ and $\Delta_{l=2} < 0$, then there exists a unique $c_L^* \in (0, c_H^*)$ where $U_{l=1}(c_L^*) = U_{l=2}(c_L^*)$ such that

$$\begin{cases} c_i < c_L^* : l = 1 \\ c_L^* \leq c_i \leq c_H^* : l = 2 \\ c_i > c_H^* : l = 3 \end{cases}$$

Figure 4 illustrates the three cases. Case 1 matches the ND data, where nominators ($l = 1$) and later learners ($l = 2$) co-exist in equilibrium; the corner cases arise when the nomination fee is prohibitively high (coordination failure, no nominations) or negligible (all $l = 1$).

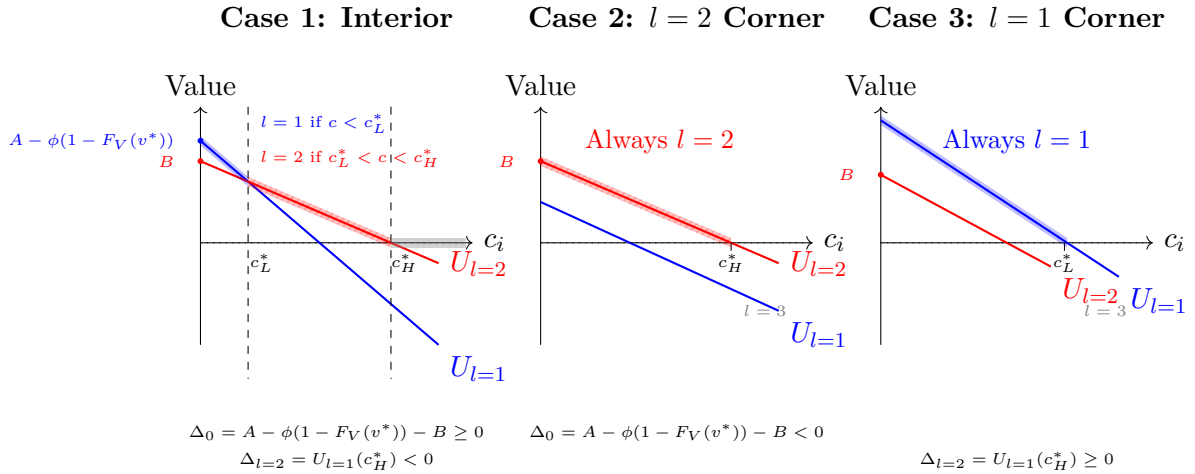


Figure 4: Equilibrium cases based on value function intercepts and slopes.

Notes: Three equilibrium cases based on the value of early learning ($U_{l=1}$) and learning later ($U_{l=2}$) as functions of learning cost c . The intersection points define cutoff costs c_L^* and c_H^* . The single crossing property ensures at most one intersection.

In Case 1, solving $U_{l=1}(c_L^*) = U_{l=2}(c_L^*)$ for the cutoff gives

$$c_L^* = \frac{A - B - \phi[1 - F_V(v^*)]}{\Pr(M < K)} \quad (14)$$

Then, the probability of a randomly arriving bidder to choose $l = 1$ is

$$\pi_0^{l=1} = \Pr[c < c_L^*] = F_C(c_L^*) \quad (15)$$

By independence of c and v , the unconditional nomination probability is the product of the two marginals:

$$\Pr[c < c_L^*, v > v^*] = \pi_0^{l=1} \cdot [1 - F_V(v^*)]$$

Similarly, the cutoff c_H^* satisfies $U(c_H^*; l = 2) = 0$, i.e. the learning cost at which the expected auction surplus of choosing $l = 2$ exactly covers the cost of learning in Stage 2:

$$c_H^* = \frac{\sum_{m=K}^{\infty} \Pr(M = m) \int_r^{\bar{v}} \Pi(v; m, \Lambda_{nn}, H_V) f_V(v | \mathcal{E}_m) dv}{\Pr(M \geq K)}. \quad (16)$$

Then probability of a random bidder choosing $l = 2$ is

$$\pi_0^{l=2} = \Pr[c_L^* < c < c_H^*] = F_C(c_H^*) - F_C(c_L^*) \quad (17)$$

Remark 1 (Post-arrivals share the later-learner cutoff: $c_{\text{post}}^* = c_H^*$). A post-arrival learns ($d = 1$) when her learning cost is below the cutoff c_{post}^* that solves $U_{\text{post}}(c_{\text{post}}^*) = 0$ in equation (8). Multiplying (8) through by $\Pr(M \geq K)$ recovers the later-learner value, $U_{l=2}(c_i) = \Pr(M \geq K) U_{\text{post}}(c_i)$: the two are proportional and therefore share the same zero. Since c_H^* solves $U_{l=2}(c_H^*) = 0$, the post-arrival's cutoff coincides with it, $c_{\text{post}}^* = c_H^*$.

4.4.1 Competition Structure

The probabilities $\pi_0^{l=1}$ and $\pi_0^{l=2}$ determine the distribution of competitors that any individual bidder expects to face.

Competition from Other Nominators Given nomination probability $\pi_0^{l=1}$, the number of other nominators M_{-i} follows from the Poisson arrival process.

Lemma 1 (Poisson Arrivals of Nominators). *If bidders arrive according to Poisson(λ_{pre}) and each independently choose $l = 1$ and then $n = 1$, meaning the probability of such event*

happening is $\pi_0^{l=1} \cdot [1 - F_V(v^*)]$, then the number of competing nominators is Poisson with the nominator rate Λ_{nom} :

$$M_{-i} \sim \text{Poisson}(\Lambda_{\text{nom}}), \quad \Lambda_{\text{nom}} := \lambda_{\text{pre}} \cdot \pi_0^{l=1} \cdot [1 - F_V(v^*)] \quad (18)$$

Proof. See Appendix D.6. □

Competition from Non-nominators The expected competition from Stage 1 arrivals choosing $l = 2$ and Stage 2 arrivals choosing $d = 1$ is given by the following two components:

$$\Lambda^{l=2} = \lambda_{\text{pre}} \cdot \pi_0^{l=2} \cdot [1 - F_{\text{post}}(r)], \quad (19)$$

$$\Lambda^{d=1} = \lambda_{\text{post}} \cdot (\pi_0^{l=1} + \pi_0^{l=2}) \cdot [1 - F_{\text{post}}(r)], \quad (20)$$

$$\Lambda_{\text{nn}} = \Lambda^{l=2} + \Lambda^{d=1}, \quad (21)$$

where Λ_{nn} is the *non-nominator competition rate* (Stage-1 later learners plus Stage-2 post-nomination arrivals), and

$$F_{\text{post}}(r) = \sum_{m=K}^{\infty} F_{V_i|\mathcal{E}_m}(r) \Pr(M = m \mid M \geq K) \quad (22)$$

is the reserve-clearing CDF averaged over the posterior $F_{V_i|\mathcal{E}_m}$ of (2).

All non-nominators enter the auction when $v \geq r$. Because they act only after observing the auction triggered ($M \geq K$), correlation shifts their own-value belief upward, so their reserve-clearing probability is the posterior $[1 - F_{\text{post}}(r)]$ of (22) rather than the prior $[1 - F_V(r)]$. Post-nomination arrivals face the same cost cutoff c_H^* (Remark 1), so the fraction that finds learning profitable at all and therefore learns, either early ($l = 1$) or later ($l = 2$) is $F_C(c_H^*) = \pi_0^{l=1} + \pi_0^{l=2}$.

4.4.2 Fixed Point Equilibrium Characterization

Consistency Requirement The payoff functions $U_{l=1}(c)$ and $U_{l=2}(c)$ depend on the expected competition $(M_{-i}, \Lambda_{\text{nn}})$, which depends on the probabilities $(\pi_0^{l=1}, \pi_0^{l=2})$, which depend on the payoff functions through equations (14), (15) and (16), (17). An equilibrium requires these objects to be mutually consistent.

Definition 1 (Equilibrium Consistency). *Probabilities $(\pi_0^{l=1}, \pi_0^{l=2})$ are **consistent** if*

1. Given $(\pi_0^{l=1}, \pi_0^{l=2})$, expected competition is characterized by $M_{-i} \sim \text{Poisson}(\Lambda_{\text{nom}})$ and Λ_{nn} , both of which can be computed.
2. Given expected competition, $U_{l=1}(c)$, $U_{l=2}(c)$ and $U_{\text{post}}(c)$ can be rewritten via equations (11), (7) and (8)
3. Given payoff functions, the optimal threshold strategy cutoff points defining l, n, d can be derived.
4. The optimal strategies generate probabilities $(\pi_0^{l=1}, \pi_0^{l=2})$ via equations (15) and (17) that equal the original probabilities

Putting everything together, a symmetric equilibrium is defined as the following.

Definition 2 (Symmetric PBE). *A symmetric equilibrium is a tuple $(v^*, c_L^*, c_H^*, \pi_{l=1}, \pi_{l=2})$ such that*

1. **Nomination threshold.** $v^* = \max\{v^{**}, r\}$, where v^{**} is the unique solution to

$$\sum_{m=K-1}^{\infty} \Pr(M_{-i} = m) \Pi(v^{**}; m, \Lambda_{\text{nn}}, H_V) = \phi. \quad (23)$$

2. **Cost cutoffs.** c_L^* solves $U(c_L^*; l=1) = U(c_L^*; l=2)$, and c_H^* solves $U(c_H^*; l=2) = 0$.
3. **Strategy probabilities.**

$$\pi_{l=1} = F_C(c_L^*), \quad \pi_{l=2} = F_C(c_H^*) - F_C(c_L^*). \quad (24)$$

4. **Competition rates (consistency).**

$$M_{-i} \sim \text{Poisson}(\lambda_{\text{pre}} \pi_{l=1} [1 - F_V(v^*)]), \quad (25)$$

$$\Lambda_{\text{nn}} = \lambda_{\text{pre}} \pi_{l=2} [1 - F_{\text{post}}(r)] + \lambda_{\text{post}} (\pi_{l=1} + \pi_{l=2}) [1 - F_{\text{post}}(r)]. \quad (26)$$

The equilibrium is a fixed point: $\pi_{l=1}$ and $\pi_{l=2}$ determine the competition rates via (25)–(26), which determine the value functions $U_{l=1}(c), U_{l=2}(c)$, determining the cutoffs c_L^*, c_H^* and recovering $\pi_{l=1}, \pi_{l=2}$ via (24).

4.5 Expansion to Heterogeneous Game

Section 4 characterizes equilibrium behavior for a single tract. In the data, tracts differ in size, royalty rate, proximity to producing wells, and the timing of nearby drilling activity

(“spud date”). These are the variables that ND’s tract listings advertise to every potential bidder before nomination begins, and they visibly correlate with bids (Table 14). Let X_t denote this vector. Although X_t is public, what it implies for a specific firm’s payoff depends on the firm’s drilling technology, machinery, and existing lease portfolio; translating X_t into a firm-specific dollar value is the costly step modeled by the learning cost c_i (Section 2).²⁶

The observable value multiplier s_t . I summarize the public, tract-level effect of X_t on values with a single scalar, $s_t \equiv s(X_t)$, which I call the tract’s *observable value multiplier*. Each bidder’s value is then

$$V_{it} = s_t \cdot \tilde{V}_{it}, \quad (27)$$

s_t is common across all bidders, raising or lowering all bidders’ values for tract t in the same proportion. $\tilde{V}_{it}$, bidder i ’s homogenized value for tract t , is the private part of her value that remains once s_t is factored out. $\tilde{V}_{it}$ is distributed according to a tract-invariant $H_{\tilde{V}}$ as in Section 4, again allowing for correlation.²⁷ Because X_t is public, every bidder observes it upon arrival and there is nothing to learn about it. The correlation that makes a nomination informative lies instead in the homogenized values. Without this normalization, treating tracts with very different X_t as though their values were on the same scale would conflate private heterogeneity across bidders with public differences across tracts.

Unlike a common value in Milgrom and Weber [1982], s_t is known to every bidder on arrival, and $\tilde{V}_{it}$ is a value rather than a signal: once learned, it is never revised in light of other bidders’ information. The only updating in the model is the revision, before she learns, of a bidder’s beliefs about her own value $\tilde{V}_{it}$ upon observing nominations (equations (1)–(2)), the defining feature of a correlated *private* values model.

Implications for the game. Values scale with s_t , but the nomination fee ϕ and the reserve r are fixed in dollars by the state, and the learning cost (land men’s time, third-party consulting fees) does not rise or fall with how lucrative a tract is.²⁸ The equilibrium objects of Section 4.4 (v^*, c_L^*, c_H^*) therefore do not simply rescale across tracts and must in

²⁶Phone interviews with bidders indicate that X_t is necessary, but far from sufficient: most areas “have been explored” in the sense that their basic geological and locational characteristics are known, but “value depends on firm-specific tech, machinery, [and] portfolio constraints.”

²⁷Formally, $H_{\tilde{V}}$ in Section 4 is the distribution of $s_t \cdot (\tilde{V}_{1t}, \dots, \tilde{V}_{Nt})$ induced by $H_{\tilde{V}}$; the equilibrium characterization of Section 4.4 carries over unchanged once restated in terms of $\tilde{V}$.

²⁸A flat \$10 nomination fee or a \$2 per-acre reserve is a rounding error on a highly promising tract but can be a meaningful deterrent on a low-value one.

general be solved tract by tract. Section 5.1 estimates s_t by a hedonic regression on X_t , and Proposition 3 shows that solving the model in residual units $\tilde{V} = V/s_t$ is equivalent to solving the dollar-denominated model tract by tract, provided the fixed-dollar instruments are rescaled in tandem ($\tilde{\phi}_t = \phi/s_t$, $\tilde{r}_t = r/s_t$, $\tilde{c} = c/s_t$). Any two tracts with the same s_t therefore generate the same residual-space equilibrium.

5 Estimation

The model is estimated in two steps. I parameterize the joint value distribution as

$$H_V(v_1, \dots, v_N) = C(F_V(v_1), \dots, F_V(v_N); \rho),$$

where $C(\cdot; \rho)$ is a Gaussian equicorrelation copula with parameter $\rho \in [0, 1)$. **Step 1** recovers the private-value distribution $\hat{F}_{\tilde{V}}$ and the copula parameter $\hat{\rho}$ from observed auction bids. **Step 2** estimates the nomination-stage structural parameters (Poisson arrival rates $(\lambda_{\text{pre}}, \lambda_{\text{post}})$ and the learning-cost distribution $F_C(\cdot; \xi_c, \omega_c, \alpha_c)$, a skew-normal with location ξ_c , scale ω_c , and shape α_c), using generalized method of moments (GMM) with the equilibrium conditions imposed as explicit constraints, that is, mathematical programming with equilibrium constraints (MPEC). Because each stage is normalized to one unit of time (Section 4), λ_{pre} and λ_{post} are the average numbers of bidders who arrive at a tract before and after nomination, so I report them as average arriving bidder counts. Full estimation details are in Appendix B.

5.1 Step 1: Bid Homogenization and Value Distribution Recovery

Raw bids b_{istc} vary multiplicatively with observable tract characteristics (size, royalty rate, well proximity). I estimate a log-linear hedonic regression to isolate the private-value component, recovering each tract's observable value multiplier $s_t = \exp(\hat{\alpha}_s + \hat{\alpha}_c + \hat{\beta}'X_t)$ and the homogenized bids $\tilde{b}_{istc} = b_{istc}/s_t$. Regression estimates are in Table 14 (reproduced in Appendix B.1); residual diagnostics supporting the log-linear specification are in Appendix B.11. By Proposition 3, the model can be solved in these residual units once the fixed-dollar primitives are rescaled: (ϕ, r, F_C) become $(\tilde{\phi}_t, \tilde{r}_t, F_{\tilde{C}_t}) = (10/s_t, r/s_t, F_{C/s_t})$ (Section 4.5). For computational simplicity, I group the 728 estimation auctions into 19 bins of s_t (Section 5.2; Appendix B.6). Because s_t is the part of a tract's value predicted by its public characteristics, I refer to tracts with a high s_t (close to existing, recently drilled wells, large) as more promising and to tracts with a low s_t (far from existing wells, small) as less promising. Bins run from the most promising (Bin 1) to the least promising (Bin 19).

Given the homogenized residual-unit bids and cross-auction variation in bidder counts I_r , I jointly estimate a monotone B-spline $\hat{F}_V$ and the equicorrelation Gaussian copula parameter $\hat{\rho}$, matching the theoretical order-statistic CDF (Theorem 1) to empirical Weibull ranks on a 200-point grid over 100 random train/test splits. The Gaussian copula achieves the lowest hold-out root mean squared error (RMSE) (Table 16; $\hat{\rho} = 0.8605$) and is also the theoretically appropriate choice for a correlated private-values setting. A lognormal tail, fitted to the B-spline and refined by a truncation-corrected maximum-likelihood step, extrapolates F_V below the observed bid support. Details are in Appendix B.3.

5.2 Step 2: Nomination-Stage Parameters

GMM moments. I match four moment conditions per bin to model-implied values: (1) the auction trigger rate $\Pr(M_b \geq K)$; (2) mean nominations per triggered tract (zero-truncated Poisson mean); (3) the variance of nominations per triggered tract (zero-truncated Poisson variance); and (4) mean total bidders per triggered auction $\mathbb{E}[I \mid M \geq K]$. Moment (1) is matched in log scale. Full definitions are in Appendix B.5, and Figure 12 shows the fit bin by bin.²⁹

Binning. Because the residual-space fee, reserve, and cost vary with s_t , the equilibrium indifference conditions, and therefore (v^*, c_L^*, c_H^*) , differ across auctions. I therefore estimate bin-specific equilibrium objects $(v_b^*, c_{L;b}^*, c_{H;b}^*)$, using each bin’s median multiplier $s_b = \text{median}_{t \in b}(s_t)$ (e.g., the fee becomes $\phi_b = 10/s_b$), while the cost distribution F_C remains shared across all bins. Bin construction details are in Appendix B.6.

MPEC implementation. Following Su and Judd [2012], I replace the nested fixed point [Rust, 1987] with MPEC: the 62-element decision vector

$$\mathbf{z} = (\lambda_{\text{pre}}, \lambda_{\text{post}}, \xi_c, \omega_c, \alpha_c, v_1^*, \dots, v_{19}^*, c_{L,1}^*, \dots, c_{L,19}^*, c_{H,1}^*, \dots, c_{H,19}^*)$$

is optimized subject to 57 equality constraints (three per bin for 19 bins, namely nomination threshold $h_{1,b}$, early learning cutoff $h_{2,b}$, and non-nominator cutoff $h_{3,b}$). Full MPEC implementation details are in Appendix B.8.

²⁹The trigger rate links each auctioned tract to the per-season available-tract universe by a fuzzy match on legal descriptions, which links 746 of the 878 auctions, so Moment (1) is undercounted by $\approx 15\%$ (Moments (2)–(4) do not depend on this match). Recovering the unmatched auctions would raise $\hat{\lambda}_{\text{pre}}$ by about 15% (to ≈ 0.006), leaving the qualitative thin-arrival conclusion and all other estimates unchanged.

6 Estimation Results

Table 2 reports the five shared structural parameters estimated by the binned MPEC procedure.³⁰ The skew-normal cost parameters are estimated directly in dollars and shared across bins; they imply a mean learning cost of $\approx \$1,161$ and a standard deviation of $\approx \$416$, and the negative shape parameter $\hat{\alpha}_c$ makes the distribution left-skewed.³¹ The mean learning cost is modest relative to the average total bid of \$11,803 across all bidders (approximately \$13,900 for winning bidders). The per-bin equilibrium objects $(\hat{v}_b^*, \hat{c}_b^{*L}, \hat{c}_b^{*H})$ are reported in Appendix Table 18; the nomination threshold $\hat{v}_b^*$ rises with the residual-space fee ϕ_b across bins. The model’s fit to the four GMM moments is reported in Appendix Table 19, and the estimated value functions $U_{l=1}(c)$ and $U_{l=2}(c)$ in Appendix B.10 (Figure 13).

The average cost of learning reflects cost-based selection into $l = 1$ versus $l = 2$. Early learners ($l = 1$) pay the least, an average of \$979; late learners ($l = 2$) pay the most, \$1,374; late (post-nomination) arrivals ($d = 1$) pay \$1,021; and the unconditional population mean is \$1,161. Nominators being the lowest-cost type and later learners the highest is precisely the sorting the model predicts: cost-efficient firms find early learning worthwhile, whereas higher-cost firms defer and wait on the information a nomination reveals.

The equilibrium cost cutoffs are estimated in residual units and convert to dollars as $\hat{c}_b^* \cdot s_b$. Evaluating the cost distribution at these dollar cutoffs shows where learning cost screens participation. In the more promising bins (bins 1–11), the dollar cutoffs sit in the far right tail of the cost distribution: the early learning ($l = 1$) share $\Pr(c < \hat{c}_b^{*L} s_b)$ is 100%, the exit share $\Pr(c > \hat{c}_b^{*H} s_b)$ is 0%, and cost imposes no screen. In the least promising, low- s bins, the dollar cutoffs fall into the body of the distribution: in the lowest- s bin the early learning share declines to 3.5% (so 96% of arrivals would not learn early) and the exit share reaches 95%. Weighted by population, about 50% of arrivals would not learn early and about 43% exit, so the model does not imply that no one exits. Cost screening is thus concentrated in the least promising tracts due to a cost-relative-to-value effect.

The learning cost cannot, however, explain the low nomination rate: even in bins 1–11, where cost imposes no screen and everyone learns upon arriving, the trigger rate is only $\approx 0.5\%$. With each stage normalized to one unit of time, $\hat{\lambda}_{\text{pre}} \approx 0.005$ is the average number of bidders who arrive at a tract during the nomination stage, so the probability that any bidder arrives at all is only $1 - e^{-0.005} \approx 0.5\%$. The low pre-nomination bidder count is therefore the root cause of the low nomination rate, while cost screening governs which tracts are nominated.

³⁰These estimates use the Joint GMM Strategy copula parameter ($\hat{\rho} = 0.861$, Table 16).

³¹Mean learning cost is calculated as $\hat{\mu}_c = \hat{\xi}_c + \hat{\omega}_c \delta \sqrt{2/\pi}$ where $\delta = \hat{\alpha}_c / \sqrt{1 + \hat{\alpha}_c^2}$

The gap between the average arriving bidder counts before nomination ($\hat{\lambda}_{\text{pre}} \approx 0.005$) and after it ($\hat{\lambda}_{\text{post}} \approx 2.894$) is consistent with an attention asymmetry: before nomination, attention is spread over the full $\approx 28,000$ -tract pool, whereas after nomination it is concentrated on the smaller set of tracts actually offered. Two patterns in the raw data support this reading. Despite $\approx 28,000$ available tracts, only ≈ 100 nominations arrive each season, and that count decreases with the size of the available pool (Figure 5(a)). Bids per auctioned tract decline as more tracts are offered (Figure 5(b)), so attention is far less diluted once nomination has pre-screened the choice set.

To quantify the dilution I fit an *attention function* to each stage—a simple relationship describing how the response received per tract falls as the number of tracts competing for attention grows:³²

$$\mathbb{E}[\text{response per candidate tract}_{sb}] = A \times (\# \text{ candidate tracts}_{sb})^{-\kappa}. \quad (28)$$

The response per candidate tract is total nominations per available tract for the nomination stage and bids per auctioned tract for the auction stage. I estimate equation (28) as a Poisson generalized linear model (GLM) at the season (s) $\times$ bin (b) level, $\text{count}_{sb} \sim \text{Poisson}(\mu_{sb})$ with $\log \mu_{sb} = \log A + (1 - \kappa) \log(\text{pool}_{sb})$, which retains bin-seasons with zero nominations. The elasticity κ measures dilution: $\kappa = 0$ means the per-tract response is independent of pool size, $\kappa = 1$ means the total response is fixed and merely spread thinner, and $\kappa > 1$ means the total response itself falls as the pool grows. Pooling both stages, each rate against its own candidate pool, yields $\hat{\kappa} \approx 1.15$ (Figure 6; robustness checks in Appendix C.1). Per-tract attention thus falls roughly as $1/\text{pool}$ —the force that keeps $\hat{\lambda}_{\text{pre}}$ orders of magnitude below $\hat{\lambda}_{\text{post}}$.

As a robustness check on the nomination-count specification, I re-estimate the model replacing the mean and variance of nominations (Moments 2–3) with the conditional multiple-nomination share $\Pr(M \geq 2 \mid M \geq 1)$. The results, and their implications for the single-rate Poisson arrival assumption, are reported in Appendix B.7.

7 Counterfactual Analysis

Nomination does four things at once. It supplies a **guaranteed bidder**: a nomination guarantees that a committed bidder is present and has paid to learn the tract, whereas an

³²Coey et al. [2020] similarly model eBay participation as a Poisson process thinned by an attention probability; the attention function here lets the thinning depend on the size of the candidate pool. It is a reduced-form, market-level relationship and differs from the individual-level “attention function” of Gabaix [2014], which maps the importance of a decision variable into the weight placed on it.

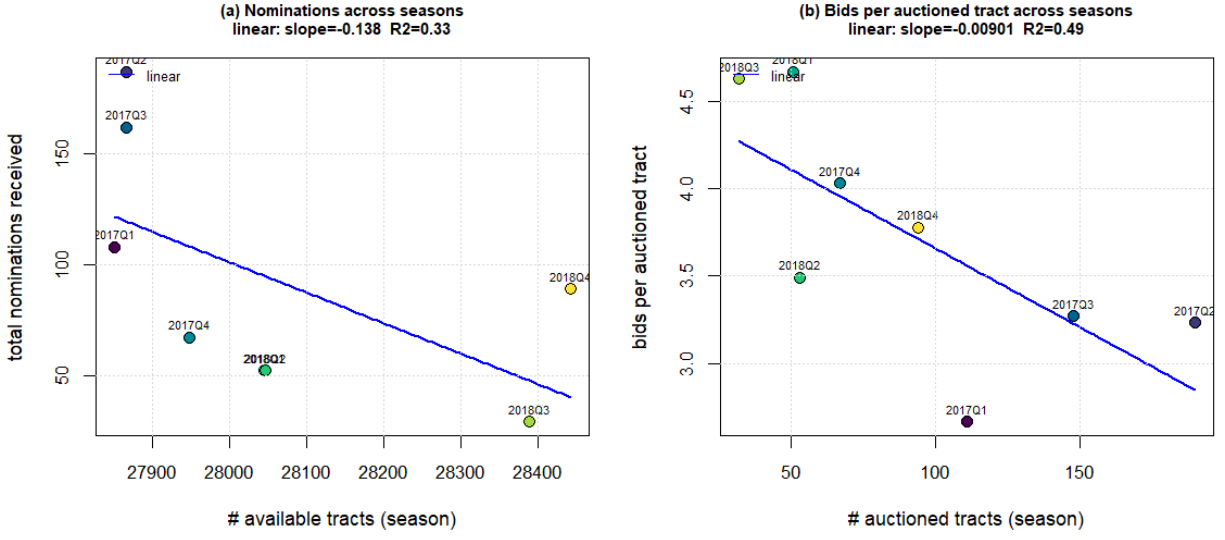


Figure 5: Attention across the two stages (raw data). (a) Total nominations received per season against the number of available tracts (one point per season, 2017Q1–2018Q4). (b) Bids per auctioned tract (total bids $\div$ tracts auctioned) against the number of tracts auctioned, one point per season.

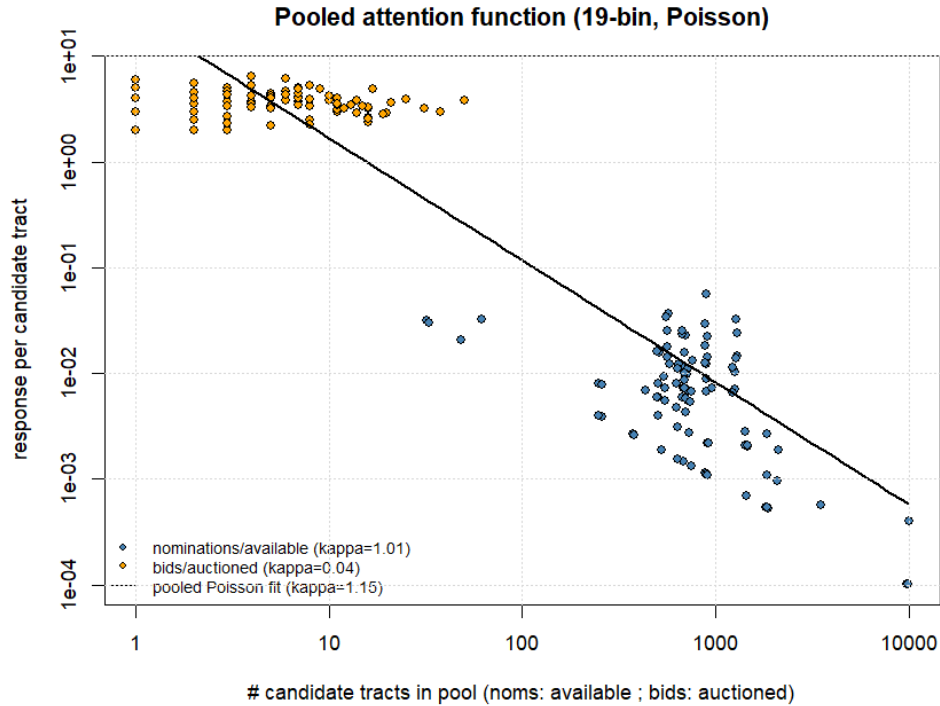


Figure 6: Pooled attention function. Per-candidate response rate against the size of the relevant candidate pool (log–log), stacking the nomination stage (total nominations per available tract vs. # available) and the bidding stage (bids per auctioned tract vs. # auctioned). The solid line is the pooled Poisson fit of equation (28), $\hat{\kappa} \approx 1.15$.

Table 2: MPEC Structural Parameter Estimates

Parameter	Description	Estimate
$\hat{\lambda}_{\text{pre}}$	Pre-auction Poisson arrival rate	0.005
$\hat{\lambda}_{\text{post}}$	Post-auction Poisson arrival rate	2.894
$\hat{\xi}_c$	Skew-Normal location (cost dist., \$)	1,712
$\hat{\omega}_c$	Skew-Normal scale (cost dist., \$)	690
$\hat{\alpha}_c$	Skew-Normal shape (cost dist.)	-126.9
	Implied SN mean (\$)	1,161
	Implied SN std. dev. (\$)	416

Notes: Binned MPEC estimation, $B = 19$ bins of s_t , break-points derived from the full available-tract universe (2017Q1–2018Q4). Each stage is normalized to one unit of time, so the Poisson arrival rates $\hat{\lambda}_{\text{pre}}$ and $\hat{\lambda}_{\text{post}}$ are the average numbers of bidders arriving at a tract in each stage. The skew-normal cost parameters ($\hat{\xi}_c, \hat{\omega}_c, \hat{\alpha}_c$) are estimated directly in dollars and shared across bins.

arbitrarily offered tract frequently draws no one. It ensures that **more valuable tracts are offered**: a firm nominates only when it has found $v \geq v^*$, so nominated tracts are more valuable than tracts the seller would pick without that signal (this selection operates on the unobservables: among tracts with the same observables, it selects those with $v \geq v^*$). Because values are correlated, a nomination creates an **information spill over**—it reveals that someone found the tract $v \geq v^*$, encouraging bidders who have not yet learned to pay to learn—but it can also cause **under-provision**, since a bidder who can wait for someone else’s nomination has less reason to learn early and nominate herself. Finally, and most importantly, offering far fewer tracts **concentrates attention**: more bidders notice and arrive at each tract on offer.

To quantify these channels I compare seven counterfactual worlds that switch off each channel one at a time, holding the estimated parameters ($\hat{\lambda}_{\text{pre}}, \hat{\lambda}_{\text{post}}, \hat{\xi}_c, \hat{\omega}_c, \hat{\alpha}_c$) fixed and re-solving each world’s equilibrium. In every world a bidder’s value for a given auction is drawn with the estimated correlation; Appendix B.13 gives the exact simulation. Beyond each party’s surplus, I track two costs that the surplus accounting does not show separately. **Wasteful learning** is the learning cost sunk by a bidder who pays to learn her value, finds it below the reserve ($v < r$), and so never bids. The **opportunity cost** of nomination is the surplus forgone on tracts that nomination leaves unoffered because the early arrival drew a value below v^* , though later arrivals might have valued them.

Worlds $\mathcal{W}_1$ – $\mathcal{W}_4$: the value of nomination holding arrival fixed. Each world removes one thing from the one before. World $\mathcal{W}_1$ is the estimated $K=1$ economy: nominated

auctions, in which the early arrival has found $v \geq v^*$ and always bids, and non-nominating entrants draw their value from the posterior correlated distribution. World $\mathcal{W}_2$ keeps the same nominated auctions but switches off the signal, the information a nomination conveys, so late entrants use the raw prior F_V —the difference $\mathcal{W}_1 - \mathcal{W}_2$ is *information spill over*. World $\mathcal{W}_3$ removes the nomination threshold: the early arrival is still present on every tract and still bids, but its value need only clear the reserve ($v \geq r$) rather than v^* —the difference $\mathcal{W}_2 - \mathcal{W}_3$ is the *selection of more valuable tracts*. World $\mathcal{W}_4$ removes the guaranteed bidder: bidders arrive at the natural rate ($\lambda_{\text{pre}} + \lambda_{\text{post}}$), and an early arrival bids only if it happens to be present, its learning cost is low enough for it to learn, and the value it then draws clears the reserve. World $\mathcal{W}_4$ is the no-nomination benchmark: the seller offers an arbitrary tract and hopes someone shows up. The successive differences sum to the value of nomination:

$$\underbrace{(\mathcal{W}_1 - \mathcal{W}_4)}_{\text{value of nomination}} = \underbrace{(\mathcal{W}_1 - \mathcal{W}_2)}_{\text{information spill over}} + \underbrace{(\mathcal{W}_2 - \mathcal{W}_3)}_{\text{more valuable tracts}} + \underbrace{(\mathcal{W}_3 - \mathcal{W}_4)}_{\text{guaranteed bidder}} .$$

I report every quantity per offered tract.³³ Table 3 and Figure 15 report the decomposition by party. Pooled across tracts, nomination raises total surplus by about \$4,239 per offered tract. The seller gains \$3,950 per offered tract, almost all of it from the guaranteed bidder (\$3,897); the selection of more valuable tracts adds only \$52, with a standard error of \$724.³⁴ Conditional on a sale, revenue rises by \$2,956, about 30% above the no-nomination benchmark, and the guaranteed bidder accounts for \$2,903 of it. Per offered tract the gain is larger, because every nominated tract sells while 10.2% of arbitrarily offered tracts in $\mathcal{W}_4$ draw no bidder. The guaranteed bidder accounts for the entire rise in the sale rate, from 89.8% to 100%, and adds 0.8 bidders per sold auction. The selection of more valuable tracts adds essentially nothing because the nomination threshold barely exceeds the reserve (v_b^*/r_b ranges from 1.03 to 1.19 across bins): a bidder who finds a tract worth bidding on is almost always willing to nominate it. What matters is that some bidder whose value clears the reserve is on the tract, not how far above the reserve that value is. On the least promising tracts (Bin 19), an arbitrarily offered tract sells only 1.4% of the time (Figure 17).

The **nominator** gains \$2,721, almost entirely from being the guaranteed bidder (\$2,718). The reason is visible in the table: in $\mathcal{W}_4$ the nominator’s surplus is only \$11.6 per tract, because without the nomination rule, she arrives at a tract the seller chose arbitrarily only

³³This is because in $\mathcal{W}_4$ a tract that draws no bidder contributes zero surplus, and since nomination’s central role is to get a committed bidder onto the tract at all, those empty tracts belong in the average.

³⁴Table 3 and Figure 15 value each bin at its median tract, which gives seller revenue over the 728 tracts of \$6.40 million without nomination and \$9.28 million with it. The abstract and the introduction value each tract at its own multiplier s_t (\$6.86 million and \$9.91 million; Appendix Table 20); the split across channels is essentially unchanged.

about 0.5% of the time: with on average $\hat{\lambda}_{\text{pre}} \approx 0.005$ bidders arriving before nomination, the probability that at least one arrives before nomination is $1 - e^{-0.005} \approx 0.5\%$. Conditional on participating, the nominator’s surplus barely changes across worlds ($\approx \$2,500$). The **non-nominators** lose \$2,432 per tract: a guaranteed strong nominator competes away the surplus they would otherwise capture on a tract with no committed buyer ($\mathcal{W}_4$ non-nominator surplus is \$7,154, the highest in the table). The externality thus runs both ways—the nominator’s presence that the seller monetizes is the same presence that competes away non-nominator rents. The selection of more valuable tracts is essentially zero for every party, and information spill over is essentially zero on average but matters for the least-promising tracts (Appendix C.7). Information spill over is small because, at the estimated nomination thresholds, almost every draw clears v^* , so observing a nomination barely changes the shape of a late entrant’s posterior (Appendices D.3 and F).

Information spill over versus under-provision. The comparison $\mathcal{W}_1 - \mathcal{W}_2$ holds nomination behavior fixed, so it measures only the spill over: how many more late arrivals learn and enter once they condition on a nomination. Pooled across tracts, the spill over raises the share of late arrivals who pay to learn their value (or ‘participation rate’) from 98.06% to 98.12% (0.1 percentage points). This effect is slightly bigger in the less promising tracts (Bins 15–19, which hold 4% of the 728 estimation tracts) where the participation rate increases from 53.0% to 54.4%, and the biggest in the least promising tract (Bin 19; the participation rate rises from 1.7% to 5.2%). The same signal could also reduce nomination itself, if bidders who expect to learn from someone else’s nomination stop learning early. To measure this *under-provision*, world $\mathcal{W}_{2b}$ switches the signal off and re-solves the early-learning cutoff $c_{L,b}^*$ in each bin, holding the nomination threshold v_b^* fixed. Under-provision is the resulting change in the probability that a tract is nominated, $(1 - F_V(v_b^*)) [F_{\tilde{C}_b}(c_{L,b}^* | \mathcal{W}_{2b}) - F_{\tilde{C}_b}(c_{L,b}^* | \mathcal{W}_1)]$. It is essentially zero: pooled across tracts, removing the signal raises the probability that an early arrival nominates only from 89.03% to 89.08% (0.04 percentage points).³⁵ The option to learn later is valuable because it avoids paying to learn on a tract that is never auctioned: bidders who learn later are waiting for the auction to be triggered, not for the information a nomination carries. Because the spill over exceeds the under-provision, the net effect of the information channel is weakly positive.

Reading learning costs off the same $\mathcal{W}_1$ and $\mathcal{W}_4$ simulations, nomination saves the resources bidders would otherwise sink into evaluating tracts along two margins.³⁷ On one

³⁵The bin-level effects are small and of mixed sign: removing the signal raises nomination in Bins 14–17 (for example, from 34.4% to 37.5% in Bin 15) and lowers it in Bins 12–13 (for example, from 56.7% to 55.0% in Bin 13), and has no effect elsewhere.

³⁷Note that these figures are already carried in the seller’s and non-nominators’ surplus from the $\mathcal{W}_1 - \mathcal{W}_4$

Table 3: Value of nomination by party and channel (\$ per offered tract; W4 counts non-entered tracts as zero; bootstrap s.e. in parentheses)

	Information spill over ($\mathcal{W}_1 - \mathcal{W}_2$)	More valuable tracts ($\mathcal{W}_2 - \mathcal{W}_3$)	Guaranteed bidder ($\mathcal{W}_3 - \mathcal{W}_4$)	Total (value of nom.) ($\mathcal{W}_1 - \mathcal{W}_4$)
Seller	1 (1)	52 (724)	3,897 (650)	3,950 (693)
Nominator	-1 (0)	4 (430)	2,718 (312)	2,721 (299)
Non-nominator	-1 (1)	25 (608)	-2,456 (638)	-2,432 (641)
Total welfare				4,239
<i>Wasted learning</i>	0 (0)	-4 (9)	-128 (12)	-132 (12)
<i>Total over the 728 tracts (\$ million)</i>				
Seller				2.88
Nominator				1.98
Non-nominator				-1.77
Total welfare				3.09

Parties sum to total welfare. The ‘Wasted learning’ row reports the change in wasted learning (learning cost sunk by bidders who draw $v < r$ and never bid). It is already netted out of the party rows and is not added to total welfare. The bottom panel multiplies the Total column by the 728 tracts in the estimation sample.

Table 4: Concentrated attention: low- vs high-attention arrival (one-stage market, \$ per tract; bootstrap s.e. in parentheses)

	Low attention ($\hat{\lambda}_{\text{low}} = 0.006$)	High attention ($\hat{\lambda}_{\text{high}} = 4.23$)	Concentrated attention (high – low)
Seller	1	8,132	8,130 (186)
Bidders	53	18,371	18,318 (823)
Sale rate	0.006	0.977	
Bidders/tract	0.01	3.91	

One-stage market without a nomination stage (Section 7), solved in equilibrium at reserve $r = \$2$. Low/high attention are the arrival rates implied by the estimated attention function spread over the $\approx 28,000$ available tracts vs. concentrated on the ≈ 73 tracts offered in a typical season.³⁶ Because $\mathcal{W}_5$ and $\mathcal{W}_6$ have no nomination stage, bidders are not split into nominators and non-nominators; the ‘Bidders’ row therefore refers to the surplus of all bidders.

hand, nomination halves *wasteful learning*, from about \$272 to \$140 per auctioned tract (‘Wasted learning’ row of Table 3). In $\mathcal{W}_1$ all of it comes from post arrivals: the nominator has already learned $v \geq v^*$ and always bids, so the only learning wasted is by post arrivals who pay to learn and draw $v < r$. They do so far less often on a nominated tract than on an arbitrary one (the probability that a late learner draws $v < r$ falls from 10.6% to 5.2%; Appendix C.8).

Almost all of the saving comes from the guaranteed bidder (\$128). In $\mathcal{W}_3$ the early arrival’s value clears the reserve, so its own learning is never wasted, and because values are correlated, post arrivals on such a tract draw $v < r$ far less often than on an arbitrary tract in $\mathcal{W}_4$, where the early arrival is usually absent and post arrivals’ values are drawn from the prior. Raising the early arrival’s value from the reserve to the nomination threshold ($\mathcal{W}_2 - \mathcal{W}_3$) saves only another \$4, and information spill over leaves wasted learning unchanged.

On the other hand, there is opportunity cost to nomination: tracts unoffered because the early arrival drew $v < v^*$ might still be valuable to later arrivals. It turns out this is not a forgone surplus. Because homogenized values are highly correlated ($\hat{\rho} = 0.861$), a low early draw likely means a low value to everyone else. The counterfactual exercise reveals auctioning such a tract would instead waste about \$3,965 of learning per tract (from about three bidders paying to learn) to realize only about \$1,571 of winning value (versus \$12,756 on a nominated tract). This nets to a negative total surplus of about $-\$2,400$ per tract, in every bin but the least promising. On these skipped tracts 54% of late learners draw $v < r$ —against about 5% on nominated tracts—so most of the learning is wasteful. The opportunity cost of the nomination screen is therefore non-positive: nomination saves learning cost both by auctioning higher-value tracts and by never convening costly search on the least promising ones.

How much would the seller need to know? The benchmark $\mathcal{W}_4$ assumes the seller has no information about which individual tracts are worth offering. A seller could instead use observable characteristics and offer tracts only from the most promising bins—those with a multiplier s_b above a cutoff—by choosing arbitrarily among them. Figure 18 plots the seller’s revenue in this seller-initiated alternative against the cutoff, holding the number of offered tracts at 728 (Table 21 gives the full breakdown). The offered tracts are spread across the included bins in the same proportions as the nominated tracts, so offering all bins reproduces the $\mathcal{W}_4$ benchmark: revenue is \$6.40 million, against \$9.28 million under nomination.³⁸ To match nomination, the seller must restrict itself to the top 8–10% (Bins 1–3) of available

difference, re-expressed in learning-cost terms.

³⁸Both totals value each bin at its median tract, as in Table 3.

tracts.³⁹⁴⁰

The revenue comparison masks three margins that move differently. On the **number of bidders**, nomination draws about 3.7 bidders per tract, against 2.6–2.8 for the seller-initiated alternative at every cutoff: the guaranteed bidder is worth one extra bidder that seller-initiation cannot reproduce. On the **sale rate**, every nominated tract sells, while the seller-initiated alternative sells 90–93%. The **price** in the seller-initiated alternative is higher: since it is very selective it offers the most promising tracts, so its average winning bid exceeds nomination’s (\$14,390 for Bins 1–3 against \$12,743), but the advantage disappears as the offered set widens (\$9,788 with all bins). A selective seller therefore beats nomination only through high prices on a few top tracts, while still fielding fewer bidders and selling a smaller share. Nomination’s value is not that it sells promising tracts at high prices—a well-informed seller could do that—but that it secures a committed bidder and a sale on every offered tract without the seller knowing which tracts are worth offering.

Removing the fee and the purchase commitment Besides the learning cost, two features of the nomination rule could limit how often tracts are nominated: the \$10 nomination fee, and the nominator’s commitment to buy the tract at the reserve if no one else bids. To measure how much each matters, I re-solve the equilibrium in every bin with each feature removed in turn and compare the trigger rate, the probability that a tract receives at least one nomination. Removing the fee raises the trigger rate, averaged over bins in proportion to their available tracts, only from 0.197% to 0.200% of available tracts, a relative increase of 1.6%.⁴¹ Removing the commitment means a nominator need not buy: a bidder who learns $v < r$ could nominate and simply walk away, so the nomination threshold becomes v^{**} instead of $v^* = \max\{v^{**}, r\}$. This leaves the trigger rate unchanged in every bin. The value at which nominating is worth the fee, v^{**} , already exceeds the reserve in every bin, so a bidder whose value falls below the reserve never nominates, and every nominated tract still sells.

³⁹Offering only these bins yields \$9.66 million, but adding Bin 4 (the top 10%) lowers revenue to \$8.98 million.

⁴⁰A seller choosing arbitrarily among all available tracts in the included bins would instead draw them in proportion to each bin’s available tracts. The break-even point does not move: offering only Bins 1–3 still beats nomination (\$9.65 million), and adding Bin 4 still falls short (\$9.07 million). What changes is the other end of the range. Because most available tracts lie in the least promising bins, offering tracts arbitrarily from all bins yields only \$1.97 million, and the sale rate falls from about 90% to 62%. A sale rate this low is not implausible: the January 2025 lease sale in Alaska’s Arctic National Wildlife Refuge offered about 400,000 acres and received no bids at all (Section 1).

⁴¹Raising the fee instead screens tracts more sharply, at the cost of a lower trigger rate (Figure 14, Appendix C.4).

Worlds $\mathcal{W}_5$ – $\mathcal{W}_6$: attention. The decomposition above holds the arrival process fixed, so it isolates what a nomination does given who shows up. To quantify the *concentrated attention* channel, I set up a one-stage auction game where bidders arrive, pay to learn their value, and compete in the auction at the reserve $r = \$2/\text{acre}$. I vary the average arriving bidder count, that is, how many bidders arrive at each offered tract (the Poisson arrival rate, with the single stage again normalized to one unit of time). In world $\mathcal{W}_5$ the average arriving bidder count is the thin attention-implied $\hat{\lambda}_{\text{low},b}$, which aggregates to 0.0058 bidders per tract: the response per available tract when bidders’ limited attention is spread over the $\approx 28,000$ available tracts (Appendix C.3). In world $\mathcal{W}_6$ that same attention is concentrated on the ≈ 73 tracts actually offered, raising the average arriving bidder count to about 4.23 bidders per auction. The difference $\mathcal{W}_5 \rightarrow \mathcal{W}_6$ measures concentrated attention.

Table 4 reports the two worlds. Under low attention the market barely functions—0.6% of tracts sell. Concentrating attention revives it completely: 98% sell and about four bidders compete per tract. The seller gains \$8,130 per tract and the bidders \$18,318. Because these worlds abstract from the nomination stage, the gains are not directly comparable to the value of nomination in $\mathcal{W}_1$ – $\mathcal{W}_4$, but they show that bidders’ attention, not the cost of learning or the nomination mechanism itself, is the binding constraint on the nomination rate.

8 Conclusion

This paper provides the first structural analysis of pre-auction nomination, quantifying four channels through which it can create value: a guaranteed bidder, the selection of more valuable tracts, an information spill over, and concentrated attention. Using novel detailed data from ND state mineral rights auctions during 2017–2018, I develop and estimate a two-stage game-theoretic model where bidders decide whether to learn early and nominate tracts or learn later after others’ nominations.

The empirical analysis reveals three key findings. First, I document that most bidders participate without nominating: while auctions attract multiple bidders, the vast majority are triggered by a single nominator. Second, using a Gaussian equicorrelation copula I find strong positive dependence in bidders’ valuations even after controlling for observable tract characteristics ($\hat{\rho} = 0.861$), indicating that common geological and market uncertainties create correlated values across competitors. Third, I estimate a left-skewed learning cost distribution (Skew-Normal with $\hat{\alpha}_c = -126.9$) with a modest mean of approximately \$1,161, small relative to the average bid of \$11,803, and standard deviation \$416 (Table 2). Learning cost screens participation only in the least promising bins: the early learning share is 100% in the more promising bins but falls to $\approx 3.5\%$ in the least promising bin, so cost governs

which tracts are nominated rather than the overall rate. The low observed nomination rate is instead a consequence of scarce early arrivals: with each stage normalized to one unit of time, on average only $\hat{\lambda}_{\text{pre}} \approx 0.005$ bidders arrive at a tract before nomination, orders of magnitude fewer than the 2.9 who arrive after it, so few firms arrive early enough to nominate even though learning early and nominating is individually worthwhile.

Compared with a no-nomination benchmark, the value of nomination rests mainly on the guaranteed bidder. Across the 728 tracts in the estimation sample, nomination raises seller revenue by about \$3.05 million, or 44% (from \$6.86 million to \$9.91 million): every nominated tract sells, and revenue conditional on a sale rises by about 30%. The selection of more valuable tracts adds essentially nothing, and the information spill over is negligible except on the least promising tracts. A seller without nomination could match its revenue only by knowing which tracts are the top 8–10% of available tracts by observable value, and even then would field fewer bidders and leave more tracts unsold. The largest lever, however, is attention: concentrating bidders' limited attention on the few nominated tracts raises the sale rate from under 1% to 98% and is worth about twice the seller's entire value of nomination (\$8,130 versus \$3,950 per tract), although the two exercises are not directly comparable because the attention worlds abstract from the nomination stage.

Nomination also economizes on the learning costs that bidders sink. On the tracts it offers, the learning wasted by bidders who learn only to find $v < r$ halves. The tracts it leaves unoffered are not a forgone opportunity: because values are strongly correlated, auctioning them would spend about \$3,965 on learning to realize about \$1,571 of value, so the opportunity cost of the nomination screen is non-positive.

Three design questions that this framework can address, but that I leave for future work, concern how the nomination rule itself is set. The first is whether the seller should raise the nomination fee to screen tract quality: a higher fee raises the nomination threshold v^{**} and selects better tracts, but taxes the decision to learn early and nominate, lowering the auction trigger rate. The second is the choice between a nomination fee and a reserve price, which screen at different stages: once the fee exceeds the maximum attainable auction surplus no firm nominates and the market collapses, whereas the reserve preserves this extensive margin but becomes less reliable when the market is thin and running an auction is costly. The third is whether requiring more nominations before initiating an auction would improve outcomes. With only about 0.005 bidders arriving at a tract before nomination, two or more coincident nominations essentially never occur, so almost no tract would be auctioned; whether this survives once firms' arrival and early-learning decisions respond to the nomination requirement change is an open question.

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A Data Summary Statistics

Tables 5 and 6 report summary statistics for per-acre and total bid values. The bid values exhibit large variation and are harmonized via the homogenization regression described in Appendix B.1.

Table 5: Bid Value (Per Acre) Summary

Variable	Average	Std Dev	Max	Min
Overall (\$)	157	752	18,001	2
Highest Bid (\$)	170	829	18,001	2
Excl. Winner (\$)	151	712	18,000	2

Notes: Summary statistics for per-acre bid values in dollars. The highest bid represents the highest bid observed in each auction, which is what the winner has to pay. “Excl. Winner” excludes the winning bid, whose recorded value reflects the ascending price proxy bidding mechanism rather than the winner’s true valuation.

Table 6: Bid Value (Total) Summary

Variable	Average	Std Dev	Max	Min
Overall (\$)	12,346	44,200	1,120,320	0.02
Highest Bid (\$)	13,874	48,372	1,120,320	0.02
Excl. Winner (\$)	11,626	42,084	1,120,160	0.12

Notes: Summary statistics for total bid values (per-acre bid $\times$ tract size) in dollars. “Excl. Winner” excludes the winning bid, whose recorded value is mechanically determined by the ascending price proxy bidding system rather than reflecting the winner’s true valuation.

Tables 7 and 8 report parcel sizes (acres) and royalty rates by season. Royalty rates are determined by the county in which the tract is located.

Tables 9–11 report information on the nearest previously drilled well: distance to the tract centroid, start of drilling date, or ‘spud date’, in calendar terms, and spud date relative to the auction date.

Table 12 summarizes starting bids (minimum bids set by nominators) by season. Most starting bids are \$2; all auctions are modeled with $r = \$2$.

Table 7: Auction Size (acres) by Season

Season	# Auctions	Avg Tract Size (acres)	Min	Max
2017-Q1	173	151	35.5	160
2017-Q2	234	137	3.3	163
2017-Q3	166	137	0.16	311
2017-Q4	67	127	25.8	161
2018-Q1	52	141	28.6	160
2018-Q2	58	131	0.06	203
2018-Q3	29	152	80	160
2018-Q4	99	80.6	0.01	161

Notes: Average, minimum, and maximum tract sizes in acres by auction season. Tract sizes are determined by state regulations and geographic characteristics but cannot exceed 160 acres.

Table 8: Count of Royalty Rates by Season

Season	# Auctions	Royalty Rate 1/16	Royalty Rate 3/16
2017-Q1	173	51	122
2017-Q2	234	58	176
2017-Q3	166	52	114
2017-Q4	67	1	66
2018-Q1	52	25	27
2018-Q2	58	17	41
2018-Q3	29	8	21
2018-Q4	99	2	97

Notes: Distribution of royalty rates (1/16 or 3/16) by season. Royalty rates vary by county location and determine the percentage of production revenue paid to the state.

Table 9: Nearest Well Distance (miles) by Season

Season	Average (miles)	Max	Min
2017-Q1	1.55	9.12	0.0881
2017-Q2	1.15	4.08	0.0105
2017-Q3	1.18	2.63	0.129
2017-Q4	0.912	2.13	0.0475
2018-Q1	0.821	2.41	0.0355
2018-Q2	0.672	1.94	0.0665
2018-Q3	0.722	2.22	0.107
2018-Q4	2.46	4.32	0.0452

Notes: Average, maximum, and minimum distance in miles from each tract to the nearest existing well, by season. Proximity to existing wells provides information about geological characteristics and production potential.

Table 10: Nearest Well Spud Date

Season	Earliest	Oldest	Missing
2017-Q1	2012-10-14	1953-12-15	30
2017-Q2	2014-08-30	1954-05-13	23
2017-Q3	2014-01-28	1955-10-06	23
2017-Q4	2013-07-03	1963-06-17	1
2018-Q1	2014-08-07	1956-05-22	10
2018-Q2	2014-06-03	1954-02-01	5
2018-Q3	2012-03-17	1960-12-02	3
2018-Q4	2014-06-12	1955-04-01	13

Notes: Date range of spud dates (when drilling began) for nearest wells by season. Some tracts have missing spud date information for their nearest well.

Table 11: Nearest Well Spud Date (How Close)

Season	Avg (days)	Max (days)	Min (days)
2017-Q1	11,949.392	23,065	1,577
2017-Q2	11,274.384	23,000	976
2017-Q3	11,988.860	22,580	1,281
2017-Q4	14,514.848	19,867	1,588
2018-Q1	13,457.952	22,540	1,279
2018-Q2	14,075.811	23,465	1,428
2018-Q3	12,373.654	21,067	2,334
2018-Q4	7,561.453	23,230	1,608

Notes: Average, maximum, and minimum time (in days) between the nearest well's spud date and the auction date. Longer time periods indicate less recent drilling activity in the area.

Table 12: Starting Bids by Season

Season	Starting at \$2	\$3	\$4	\$21	Total Auctions
2017-Q1	171	2	0	0	173
2017-Q2	210	16	6	2	234
2017-Q3	161	5	0	0	166
2017-Q4	67	0	0	0	67
2018-Q1	52	0	0	0	52
2018-Q2	58	0	0	0	58
2018-Q3	29	0	0	0	29
2018-Q4	98	1	0	0	99
Total	846	24	6	2	878

Notes: Distribution of starting bids (minimum bids) specified by nominators. The vast majority (846 out of 878) specified a minimum bid of \$2, indicating nominators set the minimum at the lowest allowed level.

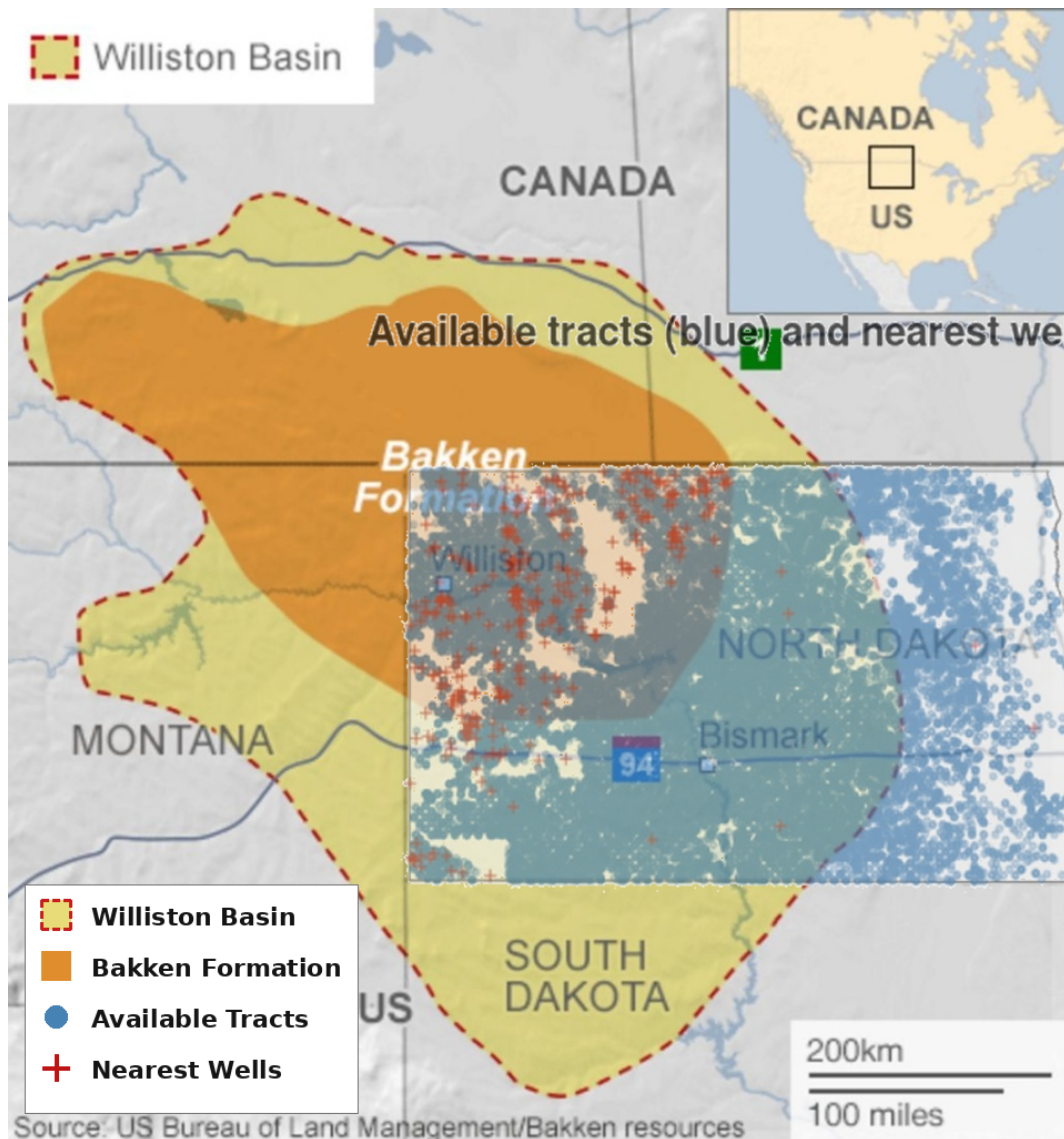


Figure 7: Available Tracts and Nearest Wells in the Williston Basin

Notes: The map overlays the boundary of the Williston Basin and the Bakken Formation with the tracts available for nomination in the estimation sample (blue) and the locations of the nearest producing wells (red crosses). Available tracts vary substantially in their proximity to an existing producing well, motivating the inclusion of distance to the nearest well and its spud date as a regressor in the bid-homogenization procedure (Appendix B.1): tracts closer to a demonstrated producer or tracts close to a recently developed well carry less geological uncertainty and are expected to command systematically higher bids, all else equal.

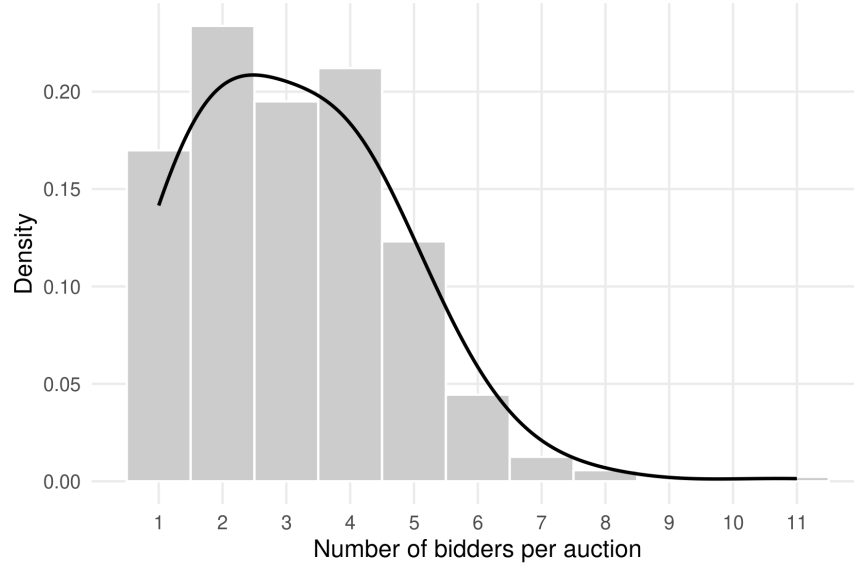


Figure 8: Distribution of the Number of Bidders per Auction

Notes: Histogram of the number of bidders per auction (one observation per auction), with a Gaussian kernel density estimate overlaid. The kernel bandwidth is $h = 2 \hat{\sigma} n^{-1/5}$, an oversmoothed Gaussian-reference choice; Silverman's rule of thumb was visibly undersmoothed for these integer counts.

Table 13: Unique Bidders per Season

Season	Unique bidders
2017-Q1	23
2017-Q2	37
2017-Q3	31
2017-Q4	19
2018-Q1	29
2018-Q2	21
2018-Q3	20
2018-Q4	23

Notes: Number of distinct bidders that submitted at least one bid in each season. Counts are per season; because many bidders are active in more than one season, the column does not sum to the overall number of distinct bidders in the sample.

B Estimation Details and Results

This appendix provides full details for the two-step estimation procedure summarised in Section 5.

B.1 Bid Homogenization

Raw bid amounts b_{istc} (bid by bidder i in auction season s for tract t in cluster c) vary multiplicatively with observable tract heterogeneity. To isolate the private-value component, I implement a two-step log-linear homogenization procedure. The key identifying assumption is an *equilibrium translational invariance* property: a uniform shift in the game-level heterogeneity Z shifts all equilibrium actions by the same amount, leaving the mapping between markup values and markup actions unchanged. Formal justification is given in Proposition 3 below, and a proof is in Appendix B.2.

I first verify that a multiplicative (log-linear) specification is appropriate. Appendix B.11 compares residual diagnostic plots for the linear and log-linear specifications. In sum, the linear model exhibits severe heteroskedasticity (residual variance fans out with fitted values) while the log-linear model stabilizes variance across the distribution. The formal model-selection rationale is discussed in the Appendix.

Appendix B.11 presents the residual diagnostic plots confirming the suitability of the log-linear specification.

Homogenization regression. I estimate the following log-linear hedonic regression:

$$\begin{aligned} \log(b_{istc}) = & \alpha_s + \alpha_c \\ & + \beta_1 \text{size}_t + \beta_2 \mathbb{I}[\text{royalty}_t = 3/16] + \beta_3 \text{distance}_t \\ & + \beta_4 \text{spudAge}_t + \beta_5 \mathbb{I}[\text{spudAge missing}] + \varepsilon_{istc} \end{aligned} \quad (29)$$

where α_s and α_c are season and cluster fixed effects (clusters are defined in Appendix B.12); size_t is tract acreage; royalty_t is the royalty rate (1/6 or 3/16); distance_t is distance to the nearest well in miles; and spudAge_t is the number of years elapsed between the spud date of the nearest well and the auction date (with an indicator for missing spud dates). Table 14 reports the estimates; the average fixed effect (approximate constant) is 7.757, and all slope coefficients carry their expected signs.

The tract's observable value multiplier is

$$s_t = \exp(\hat{\alpha}_s + \hat{\alpha}_c + \hat{\beta}' X_t) = \exp(\widehat{\log b_{istc}} - \hat{\varepsilon}_{istc}), \quad (30)$$

i.e. the exponentiated fitted value from regression (29). Homogenized bids are then the exponentiated residuals:

$$\tilde{b}_{istc} = \exp(\hat{\varepsilon}_{istc}) = \frac{b_{istc}}{s_t}. \quad (31)$$

Because every bidder in the same auction faces the same X_t , all bids in auction t are divided by the same s_t and the *relative* ranking of bids within an auction is preserved, as is the within-auction positive dependence structure arising from unobservable common shocks (geological uncertainty, commodity prices).

Descaling the nomination fee and cost. The state's flat \$10 nomination fee, $\phi = \$10$, does *not* scale with s_t . It is charged per tract regardless of its acreage or location. In residual space, this fee becomes

$$\phi = \frac{10}{s_t}. \quad (32)$$

I assume learning costs for bidder i at tract t are largely *fixed in dollar terms* and do not scale the same way as value does to observables. For example, while value decreases in distance to nearby wells, the cost of geological survey or analysis of a nearby well stays constant or, if anything, increases with distance. The estimation maintains that $C_{it} \sim \text{SN}(\xi_c, \omega_c, \alpha_c)$ is drawn from a *common* distribution across all bins so that the bin-specific residual-space distribution is $\tilde{C}_{it} = C_{it}/s_b \sim \text{SN}(\tilde{\xi}_c, \tilde{\omega}_c, \tilde{\alpha}_c)$ where $s_b > 0$.

The per-bin equilibrium structure (Appendix B.6) partially captures the resulting cross-bin heterogeneity in effective cost barriers through the bin-specific fees $\tilde{\phi}_b = \phi/s_b$.

Dollar-space *values* scale linearly with s_t , meaning auctions over a more valuable tract has proportionally larger bids, competing maxima, and surplus, so $\Pi = s_t \cdot \tilde{\Pi}$. On the other hand, the flat fee ϕ , the reserve r , and the cost distribution F_C are fixed in dollar terms and therefore do not scale. Therefore, when expressed in residual units they become auction-specific objects $\tilde{\phi}_t = \phi/s_t$, $\tilde{r}_t = r/s_t$, and $\tilde{F}_{C,t}(\tilde{c}) = F_C(s_t \tilde{c})$. Under the Skew-Normal specification, $\tilde{F}_{C,t} = \text{SN}(\xi_c/s_t, \omega_c/s_t, \alpha_c)$. The cross-auction heterogeneity in residual-space equilibria is entirely driven by this asymmetry between scaling values s_t and non-scaling parameters ϕ , r and F_C parameters. The Poisson rates Λ_{nom} and Λ_{nn} are dimensionless counts and are unaffected.

This implies two things for estimation. First, auctions sharing the same residual-space triple $(\tilde{\phi}_t, \tilde{F}_{C,t}, \tilde{r}_t)$ share the same residual-space equilibrium and may be pooled. Section B.6 groups auctions into 19 bins of s_t and solves one equilibrium per bin. Second, the dollar primitives $(\phi, r, \xi_c, \omega_c, \alpha_c)$ and the residual value distribution $F_{\tilde{V}}$, along with other structural primitives (the Poisson rates $(\lambda_{\text{pre}}, \lambda_{\text{post}})$ and the copula parameter ρ) are estimated as common to all 728 auctions. Their bin-specific residual-space counterparts are recovered

by dividing by s_b . The formal proof is in Appendix B.2.

Table 14 reports the regression results used for bid homogenization. The dependent variable is log total bid $\log(b_{istc})$. The reported constant is the average of the absorbed cluster fixed effects. The tract's observable value multiplier is $s_t = \exp(\hat{\alpha}_s + \hat{\alpha}_c + \hat{\beta}'X_t)$. Size enters as a quadratic (size + size²) to allow a concave bid-size relationship.

All slope coefficients carry their expected signs: the higher royalty rate ($3/16 > 1/16$) commands a premium; bids decline with distance to the nearest well and with the age of that well's spud date (older wells are less informative about current reservoir conditions); larger tracts attract higher total bids up to the size at which the quadratic term dominates. The spud-date missing indicator is not significant.

Table 14: Log-Bid Homogenization Regression Result

	<i>Dependent variable:</i>
	log(Total Bid)
Constant	7.757*** (0.205)
Royalty Rate (3/16)	0.324*** (0.099)
Distance to Nearest Well (miles)	−0.310*** (0.044)
Size (acres)	0.013*** (0.003)
Size ² (acres ²)	−0.0000313** (0.0000141)
Spud Date Age (years)	−0.026*** (0.002)
Observations	1,875

Notes: Bid homogenization regression (equation 29). Dependent variable is $\log(\text{per-acre bid} \times \text{acreage})$, i.e. log total bid in dollars. Standard errors in parentheses. *p<0.1; **p<0.05; ***p<0.01.

Table 15 summarizes the homogenization result. The first row reports overall log bids before homogenization ($\log b_{istc}$); subsequent rows report exponentiated residuals $\tilde{b}_{istc} =$

$\exp(\hat{\varepsilon}_{istc}) = b_{istc}/s_t$ after homogenization. Note that the winning bidders' bids are not used for this step since they are mechanically \$1 above the runner-up and do not reflect bidders' true bid. Homogenization reduces the standard deviation from 1.77 to 1.58, reflecting the removal of observable heterogeneity. The remaining dispersion captures private-value variation and unobservable common shocks.

Table 15: Homogenized Bid (Residuals)

Variable	Average	Std Dev	Max	Min
Per Acre Bid (Log Before Homogenization)	3.25	1.77	9.80	0.693
After Homogenization (Overall)	0.02095452	1.583303	5.493611	-3.973427
For Estimation	0.0305	1.59	5.49	-3.97
For Verification	-0.434	1.39	3.35	-3.74

Notes: Summary statistics of homogenized bids (residuals from the regression in Table 14). Homogenization reduces the standard deviation from 1.77 to 1.58, isolating the private value component from observable heterogeneity.

B.2 Proposition: Residual-Space Reduction

Setup. Each auction t has scaling $s_t = \exp(\hat{\alpha}_s + \hat{\alpha}_c + \hat{\beta}'X_t)$ recovered from the regression in Appendix B.1. Under private values, dollar quantities scale as $v_{jt} = s_t \tilde{v}_{jt}$ and $V_{(n:n),t} = s_t \tilde{V}_{(n:n),t}$. The fee $\phi = \$10$, reserve $\$r$, and the dollar-cost distribution $F_C \sim \text{SN}(\xi_c, \omega_c, \alpha_c)$ are common to all auctions (none scales with s_t). Define

$$\tilde{\phi}_t := \phi/s_t, \quad \tilde{r}_t := r/s_t, \quad \tilde{F}_{C,t}(\tilde{c}) := F_C(s_t \tilde{c}).$$

Under the Skew-Normal, $\tilde{F}_{C,t} = \text{SN}(\xi_c/s_t, \omega_c/s_t, \alpha_c)$: the location and scale parameters depend on s_t , while the shape α_c is invariant. This follows from the following Lemma.

Lemma 2 (Skew-normal closure under scaling). *Let $C \sim \text{SN}(\xi, \omega, \alpha)$ following the Azzalini parameterization [Azzalini, 2013, eq. 2.3], with density*

$$F_C(c) = \frac{2}{\omega} \varphi\left(\frac{c - \xi}{\omega}\right) \Phi\left(\alpha \frac{c - \xi}{\omega}\right),$$

where φ and Φ are the standard normal density and CDF. For any $s > 0$, the rescaled variable $\tilde{C} := C/s$ satisfies

$$\tilde{C} \sim \text{SN}(\xi/s, \omega/s, \alpha).$$

The location and scale parameters rescale by $1/s$ while the shape parameter α is invariant.

Proof. By the standard change-of-variables formula, $\tilde{C} = C/s$ has density

$$f_{\tilde{C}}(\tilde{c}) = s \cdot F_C(s\tilde{c}) = \frac{2s}{\omega} \varphi\left(\frac{s\tilde{c} - \xi}{\omega}\right) \Phi\left(\alpha \frac{s\tilde{c} - \xi}{\omega}\right).$$

This can be rewritten as

$$f_{\tilde{C}}(\tilde{c}) = \frac{2}{\omega/s} \varphi\left(\frac{\tilde{c} - \xi/s}{\omega/s}\right) \Phi\left(\alpha \frac{\tilde{c} - \xi/s}{\omega/s}\right),$$

which is the density of $\text{SN}(\xi/s, \omega/s, \alpha)$ by [Azzalini, 2013, eq. 2.3]. $\square$

Surplus scaling. The Gaussian equicorrelation copula depends only on ranks, so the conditional density of the maximum competing value satisfies $f_{V_{(n:n)}|v}(s_t \tilde{y}) = s_t^{-1} f_{\tilde{V}_{(n:n)}|\tilde{v}}(\tilde{y})$. Substituting $y = s_t \tilde{y}$ into the dollar-space surplus integral yields

$$\Pi(s_t \tilde{v}; n, \rho) = \int_{\tilde{r}_t}^{\tilde{v}} \underbrace{s_t(\tilde{v} - \tilde{y})}_{v-y} \underbrace{s_t^{-1} f_{\tilde{V}_{(n:n)}|\tilde{v}}(\tilde{y})}_{f_{V_{(n:n)}|v}(s_t \tilde{y})} \underbrace{s_t d\tilde{y}}_{dy} = s_t \cdot \tilde{\Pi}(\tilde{v}; n, \rho), \quad (33)$$

where the residual-space surplus is $\tilde{\Pi}(\tilde{v}; n, \rho) := \int_{\tilde{r}_t}^{\tilde{v}} (\tilde{v} - \tilde{y}) f_{\tilde{V}_{(n:n)}|\tilde{v}}(\tilde{y}) d\tilde{y}$. The Jacobian from dy cancels the Jacobian from the density, leaving a single factor of s_t from the integrand.

Nomination threshold scaling. Before turning to the value functions, I establish how the nomination threshold v^* transforms under the change of variables. Recall from (9) that $v^* = \max\{r, v^{**}\}$, where v^{**} satisfies the pointwise indifference condition

$$\sum_{m=K-1}^{\infty} \Pr(M_{-i} = m) \Pi(v^{**}; m, \Lambda_{\text{nn}}, \rho) = \phi. \quad (34)$$

By the surplus scaling (33), $\Pi(s_t \tilde{v}; m, \cdot) = s_t \cdot \tilde{\Pi}(\tilde{v}; m, \cdot)$. Substituting $v^{**} = s_t \tilde{v}_0$ into (34) and dividing both sides by s_t :

$$\sum_{m=K-1}^{\infty} \Pr(M_{-i} = m) \tilde{\Pi}(\tilde{v}_0; m, \Lambda_{\text{nn}}, \rho) = \tilde{\phi}_t. \quad (35)$$

By definition, $\tilde{v}^{**} = \tilde{v}_0$, that is, $\tilde{v}_0 = v^{**}/s_t$ satisfies the residual-space pointwise indifference condition with $\tilde{\phi}_t = \phi/s_t$. Therefore, $\tilde{v}^{**} = v^{**}/s_t$.

Next, since $\tilde{r}_t = r/s_t$,

$$\tilde{v}_t^* := \max\{\tilde{r}_t, \tilde{v}^{**}\} = \max\{r/s_t, v^{**}/s_t\} = (1/s_t) \max\{r, v^{**}\} = v_t^*/s_t.$$

The nomination threshold scales as $\tilde{v}_t^* = v_t^*/s_t$.

Value-function scaling. With the threshold scaling established, the value functions transform as follows. The dollar-space early learner value is $U_{l=1}(c) = -c - \phi[1 - F_V(v^*)] + \sum_m \Pr(M_{-i} = m) S_{l=1}(m)$, where $S_{l=1}(m) = \int_{v^*}^{\tilde{v}} \Pi(v; m + \Lambda, \rho) f_V(v) dv$. Substituting $v = s_t \tilde{v}$, $v^* = s_t \tilde{v}^*$, $f_V(v) = s_t^{-1} f_{\tilde{V}}(\tilde{v})$, and using the surplus scaling (33):

$$\begin{aligned} S_{l=1}(m) &= \int_{\tilde{v}^*}^{\tilde{v}/s_t} \underbrace{s_t \tilde{\Pi}(\tilde{v}; \cdot)}_{\Pi} \underbrace{s_t^{-1} f_{\tilde{V}}(\tilde{v})}_{f_V} \underbrace{s_t d\tilde{v}}_{dv} \\ &= s_t \int_{\tilde{v}^*}^{\tilde{v}/s_t} \tilde{\Pi}(\tilde{v}; \cdot) f_{\tilde{V}}(\tilde{v}) d\tilde{v} = s_t \cdot \tilde{S}_{l=1}(m). \end{aligned} \quad (36)$$

Therefore

$$U_{l=1}(c) = s_t \left[- \underbrace{\frac{c}{s_t}}_{\tilde{c}_t} - \underbrace{\frac{\phi}{s_t}}_{\tilde{\phi}_t} [1 - F_{\tilde{V}}(\tilde{v}^*)] + \sum_m \Pr(M_{-i} = m) \tilde{S}_{l=1}(m) \right],$$

and dividing through by s_t defines the residual-space value function

$$\tilde{U}_{l=1}(\tilde{c}_t) := \frac{U_{l=1}(c)}{s_t} = -\tilde{c}_t - \tilde{\phi}_t [1 - F_{\tilde{V}}(\tilde{v}^*)] + \sum_m \Pr(M_{-i} = m) \tilde{S}_{l=1}(m). \quad (37)$$

The identical calculation for the late-learning value yields

$$\tilde{U}_{l=2}(\tilde{c}_t) := \frac{U_{l=2}(c)}{s_t} = \Pr(M \geq K) \left[\mathbb{E}[\tilde{S}_{l=2}(m) \mid M \geq K] - \tilde{c}_t \right]. \quad (38)$$

In sum,

$$U_l(s_t \tilde{c}; \cdot) = s_t \cdot \tilde{U}_l(\tilde{c}; \cdot), \quad l \in \{1, 2\}. \quad (39)$$

Proposition 3 (Residual-Space Reduction). *Fix auction t with scaling $s_t > 0$. The dollar-space equilibrium tuple $(v^*, c_L^*, c_H^*, \Lambda_{\text{nom}}, \Lambda_{\text{nn}})$ satisfies the equilibrium system $\mathcal{S}_t$ if and only if the candidate*

$$\tilde{v}^* := v^*/s_t, \quad \tilde{c}_L^* := c_L^*/s_t, \quad \tilde{c}_H^* := c_H^*/s_t,$$

$$\tilde{\Lambda}_{\text{nom}} := \Lambda_{\text{nom}}, \quad \tilde{\Lambda}_{\text{nn}} := \Lambda_{\text{nn}}$$

satisfies the residual-space system $\tilde{\mathcal{S}}_t$. In residual space, auctions differ only through the triple $(\tilde{\phi}_t, \tilde{F}_{C,t}, \tilde{r}_t)$; all other primitives $(\lambda_{\text{pre}}, \lambda_{\text{post}}, F_{\tilde{V}}, \rho, \alpha_c, T, K)$ are common across auctions.

Proof. $\mathcal{S}_t$ comprises the nomination threshold condition (1), the cost-cutoff indifference conditions (2)–(3), and the Poisson-rate consistency conditions (4)–(5):

$$\sum_{m=K-1}^{\infty} \Pr(M_{-i} = m; \Lambda_{\text{nom}}) \Pi(v^{**}; m, \Lambda_{\text{nn}}, \rho) = \phi, \quad v^* = \max\{v^{**}, r\} \quad (1)$$

$$U_{l=1}(c_L^*) = U_{l=2}(c_L^*); \quad U_{l=2}(c_H^*) = 0 \quad (2,3)$$

$$\Lambda_{\text{nom}} = \lambda_{\text{pre}} F_C(c_L^*) [1 - F_V(v^*)];$$

$$\Lambda = \lambda_{\text{pre}} [F_C(c_H^*) - F_C(c_L^*)] [1 - F_{\text{post}}(r)] + \lambda_{\text{post}} F_C(c_H^*) [1 - F_{\text{post}}(r)] \quad (4,5)$$

The residual-space system $\tilde{\mathcal{S}}_t$ has the same form with all quantities tilded.

Verification of (1'). By the threshold-scaling derivation above, $v^{**} = s_t \tilde{v}^{**}$ satisfies (34) if and only if $\tilde{v}^{**}$ satisfies (35), the residual-space pointwise indifference condition. The corresponding nomination threshold is $\tilde{v}^* = \max\{\tilde{r}_t, \tilde{v}^{**}\} = v^*/s_t$.

Verification of (2'), (3'). By (39), $U_l(s_t \tilde{c}) = s_t \tilde{U}_l(\tilde{c})$ for $l \in \{1, 2\}$. Evaluating (2) at $c = s_t \tilde{c}_L^*$ and dividing by s_t yields $\tilde{U}_{l=1}(\tilde{c}_L^*) = \tilde{U}_{l=2}(\tilde{c}_L^*)$. Similarly, (3) at $c = s_t \tilde{c}_H^*$ divided by s_t gives $\tilde{U}_{l=2}(\tilde{c}_H^*) = 0$.

Verification of (4'), (5'). By definition of $F_{\tilde{C}}$ as the CDF of $\tilde{C} = C/s_t$,

$$F_{\tilde{C}}(\tilde{c}_L^*) = F_C(s_t \cdot c_L^*/s_t) = F_C(c_L^*); \quad F_{\tilde{V}}(\tilde{v}^*) = F_V(v^*).$$

So the RHS of (4') equals the RHS of (4), which equals $\Lambda_{\text{nom}} = \tilde{\Lambda}_{\text{nom}}$ by construction. (5') follows analogously.

The converse direction follows by the same argument with $s_t \rightarrow 1/s_t$. $\square$

B.3 Stage 2 Estimation: Value Distribution Recovery

For simplicity, this section will work with the following data structure. The data consists of R independent replicates of auctions, each containing I_r observations. Bids observed within each replicate r is denoted as $X_1^{(r)}, X_2^{(r)}, \dots, X_{I_r}^{(r)} \sim F(x)$ and are positively correlated within auction through an exchangeable copula $C_{I_r}(\cdot; \rho)$ (the Gaussian equicorrelation copula in the main specification). I assume bids are exchangeable within each auction, consistent with symmetric bidder equilibria.

Order statistics of the bids will be denoted as $X_{(1:I_r)}^{(r)} \leq X_{(2:I_r)}^{(r)} \leq \dots \leq X_{(I_r:I_r)}^{(r)}$. Often the

highest order statistic $X_{(I_r:I_r)}^{(r)}$ is missing (censored). The goal is to estimate the dependence parameter ρ and the marginal CDF $\hat{F}_V$ jointly from the censored data (Section B.3.2), and then to extend $\hat{F}_V$ below the minimum observed bid with a lognormal tail fitted by truncation-corrected maximum likelihood. This yields a smooth $\hat{F}_V$ defined on the full support, including the region below the minimum observed bid.

B.3.1 Copula Representation

By Sklar's theorem, the probability integral transforms $U_i^{(r)} = F(X_i^{(r)})$ are marginally Uniform(0, 1) but dependent through the copula $C_{I_r}(u_1, \dots, u_{I_r}; \rho)$ where ρ is the copula parameter. The CDF of the k -th order statistic then follows from the Inclusion-Exclusion Formula, stated here in the form used for estimation. First define the key symmetry condition that makes the formula tractable.

Definition 1 (Exchangeable Copula). An N -dimensional copula C_N is **exchangeable** if it is invariant under permutation of its arguments:

$$C_N(u_1, \dots, u_N) = C_N(u_{\pi(1)}, \dots, u_{\pi(N)})$$

for all $(u_1, \dots, u_N) \in [0, 1]^N$ and all permutations π .

In the bivariate case, exchangeability reduces to symmetry of the copula: $C(u, v) = C(v, u)$ for all $(u, v) \in [0, 1]^2$, which holds if and only if the associated random variables are identically distributed and exchangeable [Nelsen, 2006, Theorem 2.7.4].

Bivariate Archimedean copulas, generated via $C(u, v) = \varphi^{-1}(\varphi(u) + \varphi(v))$, are always symmetric and hence exchangeable, since addition is commutative [Nelsen, 2006, Theorem 4.1.5].⁴²

An N -dimensional Archimedean copula is constructed by the analogous extension $c_N^*(\mathbf{u}) = \varphi^{-1}(\varphi(u_1) + \dots + \varphi(u_N))$, which is symmetric under permutations by the same argument. However, this extension yields a valid N -copula for all $N \geq 2$ if and only if φ^{-1} is completely monotonic on $[0, \infty)$ [Nelsen, 2006, Theorem 4.6.2]. The Gumbel and Clayton families used in this paper both satisfy this condition and are therefore valid and exchangeable in any dimension.

The Gaussian copula $C(u_1, \dots, u_N; \Sigma) = \Phi_\Sigma(\Phi^{-1}(u_1), \dots, \Phi^{-1}(u_N))$, by contrast, is exchangeable if and only if all off-diagonal entries of the correlation matrix Σ are equal, i.e., $\Sigma_{ij} = \rho$ for all $i \neq j$ (the equicorrelated case). For a general Σ with unequal pairwise correlations, the Gaussian copula is not exchangeable. For this reason, this paper focuses on

⁴²Bivariate Archimedean copulas are generated via $C(u, v) = \varphi^{-1}(\varphi(u) + \varphi(v))$. Swapping u and v leaves the sum $\varphi(u) + \varphi(v)$ unchanged.

equicorrelated case of the Gaussian copula.

Theorem 1 (David and Nagaraja [2004], Section 5.3, formula (5.3.1)). *Let $Y_1, \dots, Y_n$ have arbitrary joint CDF $F(y_1, \dots, y_n)$ with common marginal F . Then:*

$$F_{r:n}(y) = \sum_{j=r}^n (-1)^{j-r} \binom{j-1}{r-1} H_{j:j}(y) \quad (40)$$

where $H_{j:j}(y)$ is the sum of the CDFs of the maximum over all $\binom{n}{j}$ subsets of size j . In the exchangeable case with copula C_n , all $\binom{n}{j}$ subsets share the same joint CDF, so $H_{j:j}(y) = \binom{n}{j} C_j(u, \dots, u)$ where $u = F(y)$, giving:

$$F_{r:n}(y) = \sum_{j=r}^n (-1)^{j-r} \binom{j-1}{r-1} \binom{n}{j} C_j(u, \dots, u) \quad (41)$$

which is eq. (1.2) of Maurer and Margolin [1976], who cite it back to David (1970), formula 5.5.3.

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Each term $C_j(u, \dots, u)$ corresponds to a single evaluation of the copula diagonal $\varphi^{-1}((j)\varphi(u))$, which can be computed directly in closed form.

Validity. Exchangeability of C_N is *necessary*: without it, different subsets of size r yield different joint probabilities and the copula diagonal alone is insufficient to characterise the formula.

While the copula of order statistics differs from the copula of original unordered bids, for exchangeable copulas the Inclusion-Exclusion formula depends only on diagonal copula values, which are preserved under order statistics transformations. This allows us to estimate the dependence structure from observed losers and extrapolate to the unobserved winner.

B.3.2 Joint GMM Estimation

The two unknowns in the copula estimation step are the dependence parameter ρ and the marginal value CDF F_V . I estimate both *simultaneously* using what I call the Joint GMM Strategy: a minimum-distance estimator that matches the theoretical copula order-statistic

⁴³For comparison, in the i.i.d. case the copula is the independence copula, so $C_j(u, \dots, u) = u^j = [F(x)]^j$, and the formula reduces to

$$F_{r:n}(x) = \sum_{j=r}^n (-1)^{j-r} \binom{j-1}{r-1} \binom{n}{j} [F(x)]^j,$$

which is formula (3.4.3) of Arnold et al. [2008], p. 46. The exchangeable copula formula in Theorem 1 thus nests the i.i.d. case as a special case.

CDF to empirical Weibull ranks at every observed bid. I perform 100-fold random cross-validation, partitioning auctions (not individual bids) into 90% training and 10% test sets, fitting each copula specification on the training auctions and scoring predictive fit on the held-out auctions. I use out-of-sample root mean squared error (RMSE) on the held-out auctions to select among the Gumbel, Clayton, Gaussian, and Independence specifications.

B-spline parameterisation of F_V . I parameterise F_V as a monotone B-spline

$$\hat{F}_V(x; \beta) = B(x; \beta), \quad (42)$$

with $K = 10$ internal knots placed at quantiles of the observed bids and polynomial degree 3. Monotonicity is enforced by reparameterising the coefficient vector as cumulative softplus steps: $\beta_j = \sum_{\ell \leq j} \log(1 + e^{\phi_\ell}) / (1 + \epsilon)$, so the free parameters ϕ are unconstrained while $B(\cdot; \beta)$ is guaranteed non-decreasing and maps into $[0, 1]$. The spline basis is built on all training bids; it is identified over the observed bid support $[v^*, \bar{v}]$ but not outside it.⁴⁴

Joint sum of squared residuals (SSR) objective. Let $\hat{U}_{(k)}^{(r)} := \frac{n_{k,I_r} - r_{k,I_r}(X_{(k:I_r)}^{(r)}) + 1}{n_{k,I_r} + 1}$ denote the Weibull empirical rank of bid $X_{(k:I_r)}^{(r)}$ among all position- k bids in I_r -bidder auctions, and let $G_k(u; k, I, \rho)$ be the copula order-statistic CDF from Equation 41. The joint estimator solves

$$(\hat{\phi}, \hat{\rho}) = \arg \min_{\phi, \rho} \sum_{i \in \mathcal{G}_s} \left[G(B(x_i; \beta(\phi)); k_i, I_i, \rho) - \hat{U}_{(k_i)}^{(r_i)} \right]^2, \quad (43)$$

where $G(u; k, I, \rho)$ is the CDF of the k -th order statistic from an I -sample exchangeable copula (Equation 41), evaluated at $u = B(x_i; \beta(\phi))$, the B-spline CDF at bid x_i . $\mathcal{G}_s \subseteq \mathcal{I}_s^{\text{train}}$ is a 200-point moment grid constructed from training bids in a given train/test split s by mapping an evenly-spaced target grid over $[x_{\min}^s, x_{\max}^s]$ to the nearest observed training bids, with $\mathcal{I}_s^{\text{train}}$ indexing all bids in the training set of split s . Using 200 grid points rather than all $\sim 1,500$ training bids reduces computation by approximately $7\times$ per optimizer call while preserving dense coverage of the bid support. The B-spline basis itself is built on all training bids, and only the SSR evaluation uses the grid. Equation (43) is minimised via L-BFGS-B with multiple starting values for ρ .

Lognormal parametric tail. The B-spline $B(x; \hat{\beta})$ is identified only for $x \in [v^*, \bar{v}]$. For Stage 1 I need $\hat{F}_V$ on the full support $(0, \infty)$ —in particular the scalar $1 - \hat{F}_V(v^*)$ and the

⁴⁴In this appendix v^* denotes the lower boundary of the observed bid support, not the nomination threshold v_b^* of Section 4.4.

conditional density below v^* . I therefore fit a Lognormal($\hat{\mu}, \hat{\sigma}$) by nonlinear least squares to the spline values:

$$(\hat{\mu}, \hat{\sigma}) = \arg \min_{\mu, \sigma} \sum_{i=1}^T \left[\Phi\left(\frac{\log x_i - \mu}{\sigma}\right) - B(x_i; \hat{\beta}) \right]^2, \quad (44)$$

where T is the total number of training bids. Because bids are strictly positive and the B-spline closely tracks a lognormal shape over the observed support, the nonlinear least squares (NLS) fit is tight; the lognormal then extrapolates smoothly below v^* . The scalar $1 - \hat{F}_V(v^*) = 1 - \Phi((\log v^* - \hat{\mu})/\hat{\sigma})$ and the full $(\hat{F}_V, \hat{f}_V)$ grid are saved for use in the Stage 1 equilibrium computation.

Copula family selection. I estimate Equation (43) for four copula families—Gaussian, Gumbel, Clayton, and Independence—and select by hold-out RMSE. For each of 100 random 90/10 train/test splits, the test-set RMSE is computed by evaluating the copula diagonal $D(\hat{F}_V(x_{\max}); I_r - 1, \hat{\rho})$ for each test-auction maximum bid $x_{\max}$ in an I_r -bidder auction and comparing to its empirical Weibull rank.

Copula	$\hat{\rho}$ (mean $\pm$ SD)	RMSE (mean $\pm$ SD)
Gaussian	0.866 $\pm$ 0.096	0.183 $\pm$ 0.037
Gumbel	2.404 $\pm$ 0.661	0.210 $\pm$ 0.036
Clayton	17.31 $\pm$ 4.71	0.212 $\pm$ 0.066
Independence	—	0.364 $\pm$ 0.043

Table 16: Joint GMM copula selection: hold-out RMSE across 100 random 90/10 splits (728 auctions, $I_r \in \{2, \dots, 11\}$, 200-point moment grid). Mean $\pm$ SD across splits. Bold row is the selected specification. Full-data fit (Gaussian): $\hat{\rho} = 0.8605$; the lognormal tail, refined by truncation-corrected MLE, has $\hat{\mu} = 0.032$, $\hat{\sigma} = 1.614$ (NLS stage: -0.715 , 3.863 ; see Appendix B.3).

The Gaussian copula achieves the lowest RMSE (0.183 ± 0.037). Independence is clearly dominated (RMSE = 0.364 , $2.0\times$ larger), confirming that positive dependence is a significant feature of the bid distribution. Clayton is near its upper boundary ($\hat{\rho} = 17.31 \pm 4.71$), indicating instability rather than well-identified upper-tail dependence; Gumbel is close but inferior (0.210 vs. 0.183). I therefore select the Gaussian equicorrelation copula $C_n(u_1, \dots, u_n; \rho) = \Phi_n(\Phi^{-1}(u_1), \dots, \Phi^{-1}(u_n); \rho)$ as the preferred specification. Instead of the mean across splits

(0.866), I use the median (0.861), which is essentially identical to the full-data fit (0.8605).⁴⁵⁴⁶

Economic rationale for the Gaussian copula. The empirical selection aligns with the theoretical prior imposed by the correlated private-values (CPV) assumption. In a common-value setting, bidders effectively compete for the same underlying quantity (e.g., total recoverable reserves), so a very high value for one bidder signals that all bidders have very high values—a common signal drives valuations toward the same extreme. The positive correlation setting studied here is different: each bidder forms a genuinely private assessment shaped by their own capital costs, extraction technology, and interpretation of local geological data. Bidder values are positively correlated because all bidders face the same regional geological uncertainty and energy market conditions, but this shared uncertainty acts as a common factor that shifts the *level* of the entire value distribution rather than compressing it toward a single realization.

The Gaussian equicorrelation copula is precisely the copula implied by this one-factor structure. Under the representation $V_i = \sqrt{\rho}W + \sqrt{1-\rho}\varepsilon_i$, the common factor W captures shared market and geological conditions, while the idiosyncratic term ε_i preserves the private character of each value. A key implication is that the Gaussian copula has *zero upper tail dependence*. Observing one bidder has an extreme value provides no asymptotic information that another bidder also has an extreme value.

By contrast, the Gumbel or Clayton copula has positive tail dependence, implying that extreme valuations cluster together across bidders. That the data favor the Gaussian over the Gumbel or Clayton is therefore consistent with the APV structure: common geological and market uncertainty creates equicorrelation but not extreme-value clustering.

Lognormal tail fit and structural scalars. Fitting Equation (44) to the full training data under the selected Gaussian copula yields a first-stage NLS fit $(\hat{\mu}, \hat{\sigma}) = (-0.715, 3.863)$. This NLS projection matches the spline in shape but is not a likelihood estimate of the latent value distribution, and it is sensitive to the behavior of the spline near the lower support boundary. I therefore refine it with a *truncation-corrected lognormal MLE*. Inverting the estimated Gaussian copula at the observed order statistics recovers a sample of latent value

⁴⁵The wide spread reflects that ρ is identified by cross-auction-size variation. Appendix D.8 reports subsample checks restricting to $I_r = 3$ and $I_r \in \{3, 4\}$: in both cases all copula families produce identical hold-out RMSE (within 0.001) and parameters collapse to the independence boundary, confirming that the dependence parameter is not identified without sufficient variation in I_r . Auctions with $I_r = 2$ are excluded from these checks because a single non-winner bid contributes no information about ρ , since the copula diagonal evaluates to $D(u, 1, \rho) = u$ for any copula and any ρ for $I_r = 2$ case.

⁴⁶Monte Carlo evidence further justifies using the median: Appendix D.8 documents that the distribution of $\hat{\rho}$ across replications is right-skewed for Gumbel and Clayton families at moderate sample sizes, so the median provides a less biased summary of the typical single-run estimate than the mean.

draws $\{x_i\}_{i=1}^n$. Because values below the bid-support boundary $v^* = p_{\min}$ are never observed, these draws are left-truncated at v^* . I maximize the corresponding truncation-corrected log-likelihood

$$\ell^*(\mu, \sigma) = \sum_{i=1}^n \log \left[\frac{1}{x_i \sigma} \phi \left(\frac{\log x_i - \mu}{\sigma} \right) \right] - n \log \left[1 - \Phi \left(\frac{\log v^* - \mu}{\sigma} \right) \right], \quad (45)$$

where the second term rescales the density by the mass above the truncation point v^* . Maximizing (45) gives the final tail parameters $(\hat{\mu}, \hat{\sigma}) = (0.032, 1.614)$, which are the values used in all downstream structural computations.

The lower boundary of the observed bid support is $v^* = 0.000291$, and the fitted lognormal assigns probability essentially one to bids above this threshold ($1 - \hat{F}_V(v^*) = 1 - \Phi((\log v^* - \hat{\mu})/\hat{\sigma}) \approx 0.9999998$) indicating that v^* lies deep in the left tail of the estimated value distribution. Economically, this means virtually all bidder types draw values strictly above the cost threshold, so selection into bidding is not a quantitatively important margin in this sample. The final MLE pair $(\hat{\mu}, \hat{\sigma})$ and the scalar $1 - \hat{F}_V(v^*)$ are passed directly to the Stage 1 equilibrium computation.

Corroboration: pairwise pseudo-MLE. As a cross-check, Table 17 reports results from a two-step estimator: pairwise pseudo-MLE on position-based pseudo-observations (Weibull formula) to estimate $\hat{\rho}$, followed by copula inversion and lognormal WLS to recover F_V . The Gaussian copula again achieves the lowest hold-out RMSE (0.131), with $\hat{\rho} = 0.543$. Both estimators select the same copula family, which supports the specification choice, although the two-step estimate of the dependence level is lower ($\hat{\rho} = 0.543$ versus 0.861).⁴⁷

Method	$\hat{\rho} / \hat{\rho}$	MAE	RMSE
Gumbel (pairwise)	1.5732	0.1214	0.1506
Clayton (pairwise)	0.7581	0.1288	0.1613
Clayton (n -dimensional)	0.6462	0.1342	0.1680
Independence		0.1470	0.1789
Gaussian (pairwise)	0.5425	0.1068	0.1312

Table 17: Pairwise pseudo-MLE comparison by hold-out MAE and RMSE on auctions with at least two bidders (100 random 90/10 splits; position-based ranking, Weibull formula). MAE and RMSE are computed on lognormal CDF predictions. Bold row indicates the specification also selected by the Joint GMM Strategy (Table 16).

⁴⁷The two estimates differ because the joint GMM identifies ρ through the order-statistic CDF evaluated at the winner bid, while pairwise pseudo-MLE uses only loser bids. Both are consistent estimators of the same population parameter under the maintained copula model.

Copula Parameter Consistency. A key identification assumption is that the copula parameter ρ governing the dependence among ranked losers also governs the distribution of values of all bidders.

Formally, if the joint distribution of all I_r order statistics is

$$(X_{(1:I_r)}^{(r)}, \dots, X_{(I_r:I_r)}^{(r)}) \sim C_{I_r}(\cdot; \rho)$$

then the marginal distribution of losers

$$(X_{(1:I_r)}^{(r)}, \dots, X_{(I_r-1:I_r)}^{(r)}) \sim C_{I_r-1}^{\text{marginal}}(\cdot; \rho)$$

has the **same parameter** ρ for the Gaussian equicorrelation copula, since marginalising an equicorrelation Gaussian simply reduces the dimension while leaving ρ unchanged. This allows the joint GMM estimator to use all loser bids to identify ρ and then predict the censored winner bid using the same $\hat{\rho}$.

B.4 Computing the Conditional Density of the Maximum Order Statistic

The main challenge is computing the conditional density $f_{V_{(n:n)}|v}(y|v, \rho)$. This reflects how knowing the bidder's own value v affects her beliefs about the distribution of competitors' maximum value. Under positive dependence, these variables are not independent: a higher value v suggests competitors are also more likely to have higher values. I first derive the unconditional distribution of the maximum, then compute its density, and finally obtaining the conditional density given the bidder's value. I focus on the **Gaussian copula** specification ($\hat{\rho} = 0.861$ from the Joint GMM Strategy), which achieves the lowest hold-out RMSE (Table 16); the Clayton and Gumbel copulas are estimated for comparison in the same table.

Distribution of the Maximum Under Positive Dependence From Equation 41, setting $k = n$, the cumulative distribution function of the maximum order statistic $V_{(n:n)} = \max\{V_1, \dots, V_n\}$ among n competitors is given by

$$F_{V_{(n:n)}}(y) = C_n(F(y), \dots, F(y); \rho) \quad (46)$$

I use the n -dimensional Gaussian equicorrelation copula for C_n . $F(\cdot)$ is the marginal CDF of a single bidder's value and $\hat{F}(\cdot)$ estimated from the copula-inversion procedure

(Section B.3.2) will be used. For the Gaussian copula, the diagonal evaluates to

$$C_n(u, \dots, u; \rho) = \Phi_n(\Phi^{-1}(u), \dots, \Phi^{-1}(u); \rho), \quad (47)$$

which is computed via one-factor Gauss-Hermite quadrature (28 nodes) for $n \geq 3$ and the bivariate normal CDF for $n = 2$. The conditional density simulation uses the one-factor representation $Z_j = \sqrt{\hat{\rho}}W + \sqrt{1 - \hat{\rho}}\varepsilon_j$ as described in Section D.9.

Density of the Maximum Differentiating the CDF yields the unconditional probability density function:

$$f_{V_{(n:n)}}(y) = \frac{d}{dy} F_{V_{(n:n)}}(y) = n \Phi_{n-1}\left(\frac{(1-\rho)z}{\sqrt{1-\rho^2}}, \dots, \frac{(1-\rho)z}{\sqrt{1-\rho^2}}; \frac{\rho}{1+\rho}\right) f(y), \quad z = \Phi^{-1}(F(y)) \quad (48)$$

$f(\cdot)$ is estimated by applying the central difference method on $\hat{F}(\cdot)$ estimated with the procedure in Appendix B.3.⁴⁸

Conditional Density Given Bidder's Value Finally, the conditional density $f_{V_{(n:n)}|v}(y|v, \rho)$ is derived using the Bayes rule ($f_{V_{(n:n)}|v}(y|v, \rho) = \frac{f_{V, V_{(n:n)}}(v, y; \rho)}{f_V(v)} = \frac{f_{V, V_{(n:n)}}(v, y; \rho)}{f(v)}$). This calls for specifying $f_{V, V_{(n:n)}}(v, y; \rho)$, the joint density of one's bid and the maximum of competitors' bid. The $(n+1)$ -dimension joint distribution of $(V, V_1, \dots, V_n)$, where V is the given bidder's value and the remaining n arguments are that of the competitors, can be characterized as

$$f_{V, V_1, \dots, V_n}(v, v_1, \dots, v_n; \rho) = \frac{\partial^{n+1} F_{V, V_1, \dots, V_n}}{\partial v \partial v_1 \partial v_2 \dots \partial v_n} = c_{n+1}(F(v), F(v_1), \dots, F(v_n); \rho) \cdot f(v) \cdot \prod_{i=1}^n f(v_i) \quad (49)$$

where c_{n+1} is the $(n+1)$ -dimensional copula density.⁴⁹

I implement Monte Carlo simulation to estimate the conditional density directly from the full $(n+1)$ -dimensional Gaussian equicorrelation copula with parameter $\hat{\rho} = 0.861$. The simulation uses the one-factor representation $Z_j = \sqrt{\hat{\rho}}W + \sqrt{1 - \hat{\rho}}\varepsilon_j$, where W and ε_j are i.i.d. $N(0, 1)$, and $U_j = \Phi(Z_j)$. I draw $N_{\text{sim}} = 100,000$ such $(n+1)$ -dimensional vectors. I transform competitor uniforms $U_1, \dots, U_n$ using the truncated inverse CDF $F_{V|_{\text{trunc}}}^{-1}(u) = F_V^{-1}(F_V(v^*) + u \cdot (1 - F_V(v^*)))$, which maps $[0, 1]$ to $[v^*, \infty)$, reflecting that all auction participants had $v \geq v^*$. The own value U_0 is transformed using the full inverse CDF F_V^{-1} .

⁴⁸ $f(x) \approx \frac{F(x+h) - F(x-h)}{2h}$

⁴⁹ $F_{V, V_1, \dots, V_n}(v, v_1, \dots, v_n; \rho) = C_{n+1}(F(v), F(v_1), \dots, F(v_n); \rho)$, where C_{n+1} is a $(n+1)$ -dimensional copula and $F(\cdot)$ is the common marginal CDF.

I compute the maximum competitor value $V_{(n:n)} = \max\{V_1, \dots, V_n\}$ for each draw, retain draws where $|U_0 - F_V(v)| < \varepsilon$ (tolerance $\varepsilon = 0.02 \cdot \max(F_V(v), 1 - F_V(v))$, doubled if fewer than 100 draws survive), and estimate the conditional density from the retained draws using Gaussian KDE with Silverman bandwidth, normalized to integrate to one over $[r, v]$. More details can be found in Section D.9.

B.5 Stage 1 Estimation: GMM Moments and Identification

Having estimated $(\hat{F}, \hat{\rho})$ from Stage 2, I now estimate the nomination stage parameters, arrival rates λ_{pre} (pre-auction period) and λ_{post} (post-auction period) and the cost distribution parameters $(\xi_c, \omega_c, \alpha_c)$ of F_C . These parameters determine the equilibrium nomination probabilities $(\pi_0^{l=1}, \pi_0^{l=2})$ through the fixed point system characterized in Section 4.4. I estimate them using generalized method of moments (GMM), matching model predictions to four data moments per bin.

I specify the learning cost c_i to be drawn i.i.d. from a Skew-Normal distribution $c_i \sim \text{SN}(\xi_c, \omega_c, \alpha_c)$, with CDF $F_C(c; \xi_c, \omega_c, \alpha_c) = \Phi\left(\frac{c - \xi_c}{\omega_c}\right) - 2T\left(\frac{c - \xi_c}{\omega_c}, \alpha_c\right)$, where $T(\cdot, \cdot)$ is Owen's T -function, ξ_c is the location, $\omega_c > 0$ is the scale, and α_c controls skewness. The Skew-Normal nests the symmetric Normal ($\alpha_c = 0$) and allows for skewness in either direction; the estimate $\hat{\alpha}_c < 0$ implies a left-skewed distribution. The cost is specified and estimated in dollars, common to all tracts, because learning costs (land-men's time, consulting fees) do not scale with tract value. Exactly as the \$10 nomination fee is descaled to $\tilde{\phi}_t = 10/s_t$, the cost enters the residual-space equilibrium as $\tilde{c}_{it} = c_{it}/s_t \sim \text{SN}(\xi_c/s_t, \omega_c/s_t, \alpha_c)$ (Lemma 2). In the binned estimator there is no separate cost distribution per bin: the same dollar distribution applies to all 19 bins, and all cross-bin heterogeneity in the equilibrium cutoffs (c_{Lb}^*, c_{Hb}^*) is driven by the bin-specific residual-space fee, reserve, and cost scale ($\phi/s_b, r/s_b, c/s_b$), not by different cost distributions.

B.5.1 Equilibrium Objects

For any candidate parameter vector ψ , the inner fixed point loop solves for the equilibrium tuple $(c_L^*, c_H^*, \pi_0^{l=1}, \pi_0^{l=2})$ satisfying:

$$\pi_0^{l=1} = F_C(c_L^*; \xi_c, \omega_c, \alpha_c) \tag{50}$$

$$\pi_0^{l=2} = F_C(c_H^*; \xi_c, \omega_c, \alpha_c) - F_C(c_L^*; \xi_c, \omega_c, \alpha_c) \tag{51}$$

where c_L^* solves $U_{l=1}(c_L^*) = U_{l=2}(c_L^*)$ and c_H^* solves $U_{l=2}(c_H^*) = 0$.

Given these equilibrium objects, define the Poisson rate governing the number of nomi-

nations per tract:

$$\Lambda_{\text{nom}} := \lambda_{\text{pre}} \cdot \pi_0^{l=1} \cdot [1 - \hat{F}(v^*)] \quad (52)$$

Note that Λ_{nom} depends on $(\lambda_{\text{pre}}, \xi_c, \omega_c, \alpha_c)$ through $\pi_0^{l=1} = F_C(c_L^*; \xi_c, \omega_c, \alpha_c)$, where the equilibrium cutoff c_L^* itself depends on the full competition structure including $\Lambda^{l=2}$ and $\Lambda^{d=1}$.

The non-nominator and post-arrival competition rates are:

$$\Lambda^{l=2} := \lambda_{\text{pre}} \cdot \pi_0^{l=2} \cdot [1 - \hat{F}(r)] \quad (53)$$

$$\Lambda^{d=1} := \lambda_{\text{post}} \cdot (\pi_0^{l=1} + \pi_0^{l=2}) \cdot [1 - \hat{F}(r)] \quad (54)$$

A key structural feature is that the pre-arrival pool splits into nominators and non-nominators via *independent thinnings* of the same Poisson process. By the independence of Poisson thinnings, M and the non-nominator pool are independent random variables even though both originate from the same arrival process, because they represent disjoint outcomes for each arriving bidder. Post-nomination arrivals are exogenous and independent of M by assumption. Crucially, post-nomination arrivals face the same cost-screening decision as non-nominators: each must pay the learning cost c_i before deciding whether to bid. Since the auction surplus available to a post-period arrival is identical to that of a non-nominator (both observe the same nominators and compete in the same auction), their cost cutoff is c_H^* , identical to that of a non-nominator. Only the fraction $F_C(c_H^*; \xi_c, \omega_c, \alpha_c)$ of post-nomination arrivals find it worthwhile to pay the learning cost and enter the auction.⁵⁰

This independence means that conditional on the auction triggering ($M \geq K$), the conditional mean of total auction participants decomposes as:

$$\mathbb{E}[I \mid M \geq K] = \mathbb{E}[M \mid M \geq K] + \Lambda^{l=2} + \Lambda^{d=1} \quad (55)$$

where $\mathbb{E}[M \mid M \geq K] = \sum_{m=K}^{\infty} m \cdot \Pr(M = m) / \Pr(M \geq K)$, which for $K = 1$ simplifies to the zero-truncated Poisson mean $\Lambda_{\text{nom}} / (1 - e^{-\Lambda_{\text{nom}}})$ ⁵¹.

B.5.2 Identification Strategy

I exploit four moment conditions per bin, organized into two groups: three moments from the nomination process—Moment 1 (the auction trigger rate), Moment 2 (the mean number of nominations), and Moment 3 (the variance of nominations)—and one moment from bidder

⁵⁰See Remark 1 for the formal argument.

⁵¹The naive approximation $\mathbb{E}[I \mid M \geq K] \approx \Lambda_{\text{nom}} + \Lambda^{l=2} + \Lambda^{d=1}$ is incorrect because $\mathbb{E}[M \mid M \geq K] > \Lambda_{\text{nom}}$ for any finite Λ_{nom} , so ignoring the truncation systematically understates competition.

counts, Moment 4 (the mean number of bidders).

Moment 1: Auction Trigger Rate. The trigger rate in bin b is defined using the full universe of available tracts—not just the estimation sample—so that the denominator correctly reflects the population of tracts that *could* have been nominated:

$$\bar{p}_{\text{trigger},b} = \frac{1}{|S|} \sum_{s \in S} \frac{\text{tracts auctioned in bin } b \text{ during season } s}{\text{tracts available for auction in bin } b \text{ during season } s},$$

where S is the set of auction seasons in the sample. A tract is “available” in season s if it has not been leased within the prior five years and its current lease (if any) has expired by the auction date. Averaging over seasons reduces noise from season-specific variation in tract supply. I use the full tract universe provided through the ND Department of Trust Lands, with the same observable characteristics of tracts (size, royalty rate and nearest-well information) used to predict s_t (and hence $\phi_b = 10/s_t$) for each available tract, so that tracts outside the estimation sample can be assigned to the closest bin.

Under the model, $M_t \sim \text{Poisson}(\Lambda_{\text{nom},b})$, so the bin- b trigger probability is:

$$p_{\text{trigger},b}(\psi) = \Pr(M \geq K) = 1 - e^{-\Lambda_{\text{nom},b}} \quad (K = 1).$$

The moment condition matched in log scale is:⁵²

$$g_{1,b}(\psi) = \log \bar{p}_{\text{trigger},b} - \log p_{\text{trigger},b}(\psi) = 0. \quad (56)$$

Moment 2: Mean Nominations per Nominated Tract. Among tracts that received at least one nomination, the average number of nominations provides additional information about the nomination rate:

$$\bar{m}_{\text{nom}} = \frac{1}{|\mathcal{A}|} \sum_{t \in \mathcal{A}} M_t$$

where $\mathcal{A}$ is the set of tracts with $M_t \geq K = 1$. Under the model, conditional on $M \geq 1$, the number of nominations follows a zero-truncated Poisson distribution:

$$\mathbb{E}[M \mid M \geq 1] = \frac{\Lambda_{\text{nom}}}{1 - e^{-\Lambda_{\text{nom}}}}$$

The moment condition is:

$$g_2(\psi) = \bar{m}_{\text{nom}} - \frac{\Lambda_{\text{nom}}(\psi)}{1 - e^{-\Lambda_{\text{nom}}(\psi)}} = 0$$

⁵²Moment 1 is matched in log scale since the trigger rate is a small positive quantity.

Identification from Moments 1–2. Although Moments 1–2 are all functions of the single Poisson rate Λ_{nom} , the key insight is that $\Lambda_{\text{nom}} = \lambda_{\text{pre}} \cdot F_C(c_L^*) \cdot [1 - \hat{F}(v^*)]$ is not a free parameter but a composite function of $(\lambda_{\text{pre}}, F_C)$. The equilibrium cutoff c_L^* depends on F_C through the fixed point system. In other words, different cost distributions generate different equilibrium nomination probabilities $\pi_0^{l=1} = F_C(c_L^*)$ and different late-learning rates $\pi_0^{l=2}$, which feed back into the competition structure and therefore into c_L^* itself. Consequently, Moments 1–2 collectively pin down Λ_{nom} . The equilibrium consistency conditions imposed in the inner loop create the nonlinear mapping that separates λ_{pre} from F_C .

Moment 3: Variance of Nominations per Nominated Tract. The variance of nomination counts among nominated tracts provides additional information about the shape of the nomination distribution:

$$\widehat{\text{Var}}_{\text{nom}} = \frac{1}{|\mathcal{A}| - 1} \sum_{t \in \mathcal{A}} (M_t - \bar{m}_{\text{nom}})^2.$$

Under the zero-truncated Poisson,

$$\text{Var}(M \mid M \geq 1) = \frac{\Lambda_{\text{nom}}}{1 - e^{-\Lambda_{\text{nom}}}} \left(1 - \frac{\Lambda_{\text{nom}} \cdot e^{-\Lambda_{\text{nom}}}}{1 - e^{-\Lambda_{\text{nom}}}} \right),$$

and the moment condition is

$$g_3(\psi) = \widehat{\text{Var}}_{\text{nom}} - \text{Var}(M \mid M \geq 1; \psi) = 0. \quad (57)$$

Because Moments 1–2 already pin Λ_{nom} under the Poisson structure, Moment 3 is not independently identifying, but it is retained as an active moment (unit weight) that disciplines the concentration of the nomination distribution.

Moment 4: Mean Number of Bidders per Triggered Auction. The average number of total bidders per triggered auction from data ($\bar{I}_{\text{bid}} = \frac{1}{|\mathcal{A}|} \sum_{t \in \mathcal{A}} I_t$) identifies the overall level of competition via equation (55):

$$g_4(\psi) = \bar{I}_{\text{bid}} - [\mathbb{E}[M \mid M \geq K] + (\Lambda^{\text{pio}, \text{bid}} - \Lambda_{\text{nom}}) + \Lambda^{l=2} + \Lambda^{d=1}] = 0 \quad (58)$$

where $\mathbb{E}[M \mid M \geq K] = (\Lambda_{\text{nom}} - \sum_{m=0}^{K-1} m \cdot \Pr(M = m)) / \Pr(M \geq K)$, and $\Lambda^{\text{pio}, \text{bid}} - \Lambda_{\text{nom}}$ is the expected count of early learners who bid without nominating ($r \leq v < v^*$). This moment brings in λ_{post} through $\Lambda^{d=1}$ and $\pi_0^{l=2}$ through $\Lambda^{l=2}$. However, it identifies chiefly the *product* of arrival rate and entry probability, $\Lambda_b = \lambda \pi_b(F_C)$; the level of λ_{post} and the

cost scale are therefore only weakly separated (see the identification remark below).

Identification of the Arrival-Rate Level. Moments 1–4 identify the *products* $\Lambda_b = \lambda \pi_b(F_C)$ of arrival rate and entry probability. Consequently a high- λ /high-cost-barrier configuration (small π_b) and a low- λ /low-cost configuration (large π_b) generate nearly identical Λ_b and fit these moments comparably. Therefore, the *level* of $(\lambda_{\text{pre}}, \lambda_{\text{post}})$ relative to the cost scale is only weakly identified; the estimates in Section 6 should be read with this caveat.

B.6 Binning: Handling Variation in the Residual-Space Nomination Fee

After homogenization, the flat \$10 nomination fee becomes $\phi = 10/s_t$, which varies across auctions because s_t reflects observable tract heterogeneity. This variation means the equilibrium indifference conditions depend on the auction: a bidder with value v^* indifferent between nominating and not nominating at a high- s_t auction faces a lower residual-space fee than at a low- s_t auction. Treating all auctions as sharing a single equilibrium would therefore be incorrect.

I address this by grouping the 728 estimation auctions into 19 bins of s_b , with breakpoints derived from the full available-tract universe, i.e. all tracts available for nomination across the eight nomination seasons (2017Q1–2018Q4), rather than from the estimation sample alone. Starting from fine quantile breaks on the universe distribution of s_t , adjacent bins are iteratively merged until each bin contains at least the required minimum number of estimation auctions and the bin-average trigger rate is weakly declining in s_b where s_b is the within-bin median:

$$s_b = \text{median}_{t \in b}(s_t) \quad (59)$$

The binned MPEC estimates bin-specific equilibrium objects $(v_b^*, c_{Lb}^*, c_{Hb}^*)$ for each of the 19 bins while keeping the cost distribution $\text{SN}(\xi_c, \omega_c, \alpha_c)$ shared. All cross-bin heterogeneity in equilibrium cutoffs is therefore driven entirely by the bin-specific fees ϕ_b , not by different cost distributions.

The reason for per-bin moment matching is because nomination fee and learning costs do not scale in the same way as bids. The tract-level observable value multiplier s_t from bid homogenization should be divided from the two. As a result, the residual-space nomination fee, reserve price and cost, ϕ/s_t , r/s_t and c_i/s_t , is systematically smaller for high- s_t bins (large, high-quality tracts) and larger for low- s_t bins (small or marginal tracts). Bins therefore face genuinely different effective cost barriers, and the observable moment patterns

(trigger rates, nomination counts, bidder counts) differ across bins for this economic reason. Matching each bin to its own empirical moments allows the model to fit those cross-bin differences directly, rather than averaging them away into a single global target.

The GMM objective stacks the per-bin moment residuals into a single vector,

$$g(\mathbf{z}) = (g_{1,1}, g_{2,1}, g_{3,1}, g_{4,1}, \dots, g_{1,19}, g_{2,19}, g_{3,19}, g_{4,19})^\top,$$

where $g_{k,b}$ denotes the moment $k \in \{1, 2, 3, 4\}$ residual for bin b . Each of the 19 bins contributes these four moment conditions, resulting in 76 active moments. The weighting matrix W is diagonal, with unit weight on the trigger, mean-nomination, and variance residuals and weight 1.6 on the mean-bidder residual.

B.7 Robustness: Nomination Counts via $\Pr(M \geq 2 \mid M \geq 1)$

The baseline specification summarizes the nomination-count distribution through its trigger rate (Moment 1), conditional mean (Moment 2), and conditional variance (Moment 3), each of which is a function of the single Poisson rate Λ_{nom} . As a robustness check I re-estimate the model with an alternative characterization that keeps the trigger (Moment 1) but replaces the conditional mean and variance (Moments 2–3) with the conditional multiple-nomination share

$$\Pr(M \geq 2 \mid M \geq 1) = \frac{1 - (1 + \Lambda_{\text{nom}}) e^{-\Lambda_{\text{nom}}}}{1 - e^{-\Lambda_{\text{nom}}}}, \quad (60)$$

matched in log scale.⁵³ This moment targets the frequency of *multiple* nominations directly rather than the first two conditional moments of the count.

The re-estimated arrival rates preserve the qualitative pattern of the baseline: the pre-period arrival rate remains tiny ($\hat{\lambda}_{\text{pre}} \approx 0.0003$, against ≈ 0.005 at the baseline) and the post-period rate moderate ($\hat{\lambda}_{\text{post}} \approx 4.1$, against ≈ 2.9). However, the fit deteriorates sharply: the GMM objective rises to $\approx 2,166$, against ≈ 35 at the baseline. The reason is structural rather than numerical. Under a single Poisson rate, the trigger and the multiple-nomination share are both governed by Λ_{nom} , so matching the low observed trigger ($\Pr(M \geq 1) \approx 0.7\%$) forces $\Pr(M \geq 2 \mid M \geq 1) \approx \Lambda_{\text{nom}}/2 \approx 0.4\text{--}0.8\%$. In the data, however, the conditional multiple-nomination share averages 7.7% (and reaches 16% in some bins)—an order of magnitude larger than any Poisson consistent with the observed trigger can produce. A single-rate Poisson nomination process is therefore rejected by this moment: conditional on being nominated, tracts draw two or more nominations far more often than indepen-

⁵³The complementary single-nomination share $\Pr(M = 1 \mid M \geq 1) = 1 - \Pr(M \geq 2 \mid M \geq 1)$ carries no additional information and is not matched separately.

dent Poisson arrivals allow. This overdispersion (clustering) in nominations points toward a mixed-Poisson / negative-binomial arrival extension, in which a dispersion parameter separates the overall nomination rate from the frequency of multiple nominations.⁵⁴ I retain the Poisson baseline for the reported results and leave the overdispersed extension to future work.

B.8 MPEC Implementation

The goal is to estimate the structural primitives $\boldsymbol{\psi} = (\lambda_{pre}, \lambda_{post}, \xi_c, \omega_c, \alpha_c)$ using a GMM objective. Rather than the standard nested fixed point (NFXP) approach [Rust, 1987], I adopt the constrained optimization approach of Su and Judd [2012] (MPEC).⁵⁵ I augment the parameter vector with equilibrium objects and impose equilibrium conditions as explicit constraints, rather than solving for equilibrium at each candidate parameter value. Specifically, I solve

$$\hat{\mathbf{z}} = \arg \min_{\mathbf{z}} g(\mathbf{z})' W g(\mathbf{z})$$

$$\text{where } \mathbf{z} = \left(\underbrace{\lambda_{pre}, \lambda_{post}, \xi_c, \omega_c, \alpha_c}_{5 \text{ structural}}, \underbrace{v_1^*, \dots, v_{19}^*}_{19 \text{ bin thresholds}}, \underbrace{c_{L,1}^*, \dots, c_{L,19}^*}_{19 \text{ early learner cutoffs}}, \underbrace{c_{H,1}^*, \dots, c_{H,19}^*}_{19 \text{ } l_i = 2 \text{ cutoffs}} \right) \quad (61)$$

The decision vector $\mathbf{z}$ comprises five shared structural parameters $\boldsymbol{\psi} = (\lambda_{pre}, \lambda_{post}, \xi_c, \omega_c, \alpha_c)$ plus per-bin equilibrium objects $(v_b^*, c_{Lb}^*, c_{Hb}^*)$ for bins $b = 1, \dots, 19$, 62 elements in total.

⁵⁴Hickman et al. [2017] reach a similar conclusion for bidder arrivals at eBay auctions: fitted to the observed lower bounds on participation, a generalized Poisson model, which has a larger variance, fits clearly better than a standard Poisson.

⁵⁵See Dubé et al. [2012] for its computational advantages when estimating equilibrium models, particularly when the inner fixed point is expensive or exhibits convergence difficulties.

$$\begin{aligned}
\pi_{0,b}^{l=1} &= F_C(c_{Lb}^*; \xi_c, \omega_c, \alpha_c) \\
\pi_{0,b}^{l=2} &= F_C(c_{Hb}^*; \xi_c, \omega_c, \alpha_c) - F_C(c_{Lb}^*; \xi_c, \omega_c, \alpha_c) \\
\pi_{0,b}^{\text{bid}} &= F_C(c_{Hb}^*; \xi_c, \omega_c, \alpha_c) = \pi_{0,b}^{l=1} + \pi_{0,b}^{l=2} \\
\Lambda_b^{\text{nom}} &= \lambda_{\text{pre}} \cdot \pi_{0,b}^{l=1} \cdot [1 - F_V(v_b^*)] \\
\Lambda_b^{\text{pio,bid}} &= \lambda_{\text{pre}} \cdot \pi_{0,b}^{l=1} \cdot [1 - F_V(r_b)] \\
\Lambda_b^{FR+\text{post}} &= \lambda_{\text{pre}} \cdot \pi_{0,b}^{l=2} \cdot [1 - F_{\text{post}}(r_b)] \\
&\quad + \lambda_{\text{post}} \cdot \pi_{0,b}^{\text{bid}} \cdot [1 - F_{\text{post}}(r_b)] \\
F_{\text{post}}(r_b) &= \sum_{m \geq K} F(r_b | m) \Pr(M = m | M \geq K)
\end{aligned}$$

Learning is required for auction entry; conditional on learning, every type bids whenever $v \geq r$. Early learners (including both nominators and non-nominators) value the tract from the prior, so their reserve-clearing probability is $[1 - F_V(r)]$. Non-nominating entrants learn only after observing that the auction triggered; by positive correlation their values follow the posterior conditional on $M \geq K$, so their reserve-clearing probability is $[1 - F_{\text{post}}(r_b)]$. Accordingly Λ_b^{nom} ($v \geq v_b^*$) governs only the *trigger* probability, while the competition/bidding objects use $[1 - F_V(r_b)]$ for early learners ($\Lambda_b^{\text{pio,bid}}$, which includes the early learning non-nominators $\Lambda_b^{\text{pio,bid}} - \Lambda_b^{\text{nom}}$) and $[1 - F_{\text{post}}(r_b)]$ for late learners ($\Lambda_b^{FR+\text{post}}$). The post-period term includes $\pi_{0,b}^{\text{bid}} = F_C(c_{Hb}^*)$ since $c_{\text{post}}^* = c_H^*$. The mean bidder count per triggered auction is $\mathbb{E}[M | M \geq K] + (\Lambda_b^{\text{pio,bid}} - \Lambda_b^{\text{nom}}) + \Lambda_b^{FR+\text{post}}$.

Nominators use $[1 - F_V(v_b^*)]$ since they only bid when they nominated instead of exiting; non-nominators and post-nomination arrivals use $[1 - F_V(r_b)]$ since they bid whenever their value exceeds the auction reserve. The post-period term includes $\pi_{0,b}^{\text{bid}} = F_C(c_{Hb}^*)$ reflecting the $c_{\text{post}}^* = c_H^*$ assumption.

The optimization is subject to 57 equality constraints (three per bin: $h_{1,b}$, $h_{2,b}$, and $h_{3,b}$) and 19 inequality constraints (one per bin, $g_{1,b}$, corresponding to the participation condition $A_b - B_b \geq 0$), encoding the equilibrium conditions from Definition 2. The learning cost F_C is parameterized as a skew-normal distribution $\text{SN}(\xi_c, \omega_c, \alpha_c)$ with support on $\mathbb{R}$, so realized costs may be negative; rather than truncating the distribution at zero, the inequality constraints $g_{1,b}$ keep the equilibrium cost cutoffs nonnegative, so that only the nonnegative part of the support is economically active.

The nomination threshold v_b^* is estimated jointly with $c_{L,b}^*$ and $c_{H,b}^*$ for each bin, with $h_{1,b}$ (the indifference condition $\tilde{\Pi}(v_b^*) = \phi_b$) imposed as an explicit equality constraint.

First, the **Nomination threshold** constraint for bin b , derived from equation 23, is

characterized as

$$h_{1,b}(\mathbf{z}) = \tilde{\Pi}(v_b^*; m = K - 1, \Lambda_{\text{nnb}}, \hat{\rho}) - \phi_b = 0, \quad \phi_b = 10/\text{median}_{t \in b}(s_t) \quad (62)$$

where $\tilde{\Pi}(v_b^*; \cdot)$ is the expected auction surplus evaluated at the bin- b nomination threshold, and ϕ_b is the representative residual-space nomination fee for bin b . This ensures that a nominator with value exactly v_b^* earns auction surplus equal to bin b 's residual-space nomination fee, and is therefore indifferent between nominating and not nominating.

The constraint $h_{1,b}$ is maintained as an active equality constraint for each bin in estimation. Because $\phi_b = 10/\text{median}_{t \in b}(s_t)$ differs across bins, the equilibrium threshold v_b^* is bin-specific: bins with a higher residual-space fee require a higher value threshold to maintain the nomination indifference condition, and vice versa. This cross-bin variation in v_b^* is identified from the bin-level indifference condition $\tilde{\Pi}(v_b^*; \cdot) = \phi_b$, without imposing any additional identifying assumption on the threshold.

Next, the $l = 1$ **cutoff** constraint for bin b , derived from $U_{l=1}(c_{Lb}^*) = U_{l=2}(c_{Lb}^*)$, is

$$h_{2,b}(\mathbf{z}) = c_{Lb}^* - \frac{A_b - \phi_b[1 - F_V(v_b^*)] - B_b}{1 - \Pr(M_{-i} \geq K)} = 0 \quad (63)$$

where $A_b = \sum_{m=K-1}^{\infty} \Pr(M_{-i} = m) \cdot S_{l=1}(m)$ is the expected gross auction surplus for an early learner in bin b , $B_b = \Pr(M_{-i} \geq K) \cdot E[S_{l=2}(m) \mid M \geq K]$ is the expected gross surplus for a non-nominator, and $\phi_b[1 - F_V(v_b^*)]$ is the expected nomination fee (paid with probability $1 - F_V(v_b^*)$). This follows from setting $U_{l=1}(c_{Lb}^*) = U_{l=2}(c_{Lb}^*)$ and solving for c_{Lb}^* using equation (11). $S_{l=1}(m)$ integrates $\tilde{\Pi}$ against the full $f_V(v)$; $S_{l=2}(m)$ integrates $\tilde{\Pi}_{FR}$ against the joint Gaussian copula posterior $f(v \mid v_1 \geq v_b^*, \dots, v_m \geq v_b^*)$, precomputed on a (40×15) grid of (u^*, m) pairs and linearly interpolated at runtime.

The **Non-nominator** ($l_i = 2$) **cutoff** related constraint, derived from equation 16, is given as

$$h_3(\mathbf{z}) = c_H^* - \mathbb{E}[S_{l=2}(m) \mid M_{-i} \geq K] = 0 \quad (64)$$

where $S_{l=2}(m)$ uses the joint Gaussian copula posterior density for the non-nominator's own value, not the independently truncated marginal.

Lastly, for each bin b , two inequality constraints ensure the interior solution condition:

$$\begin{aligned} A_b - \phi_b[1 - F_V(v_b^*)] - B_b &\geq 0 \\ c_{Lb}^* &\leq c_{Hb}^* \end{aligned} \quad (65)$$

The first condition, $\Delta_{0,b} = U_{l=1}(0) - U_{l=2}(0) \geq 0$ within bin b , ensures that the zero-cost

bidder prefers early learning over late-learning in that bin, guaranteeing an interior solution with positive nominations. The second ensures the bin-level cost cutoffs are correctly ordered.

The weighting matrix is the diagonal matrix (Appendix [B.6](#)).

B.9 Per-Bin Equilibrium Objects and Moment Fit

Table 18: Per-Bin Equilibrium Objects

Bin	$\hat{v}_b^*$	$\hat{c}_b^{*L}$	$\hat{c}_b^{*H}$
1	0.0569	1.550	1.873
2	0.0798	1.545	1.914
3	0.0787	1.545	1.916
4	0.0894	1.543	1.896
5	0.0942	1.542	1.904
6	0.1224	1.536	1.965
7	0.1128	1.537	1.950
8	0.1375	1.532	1.986
9	0.1406	1.531	1.990
10	0.1293	1.532	1.975
11	0.2588	1.514	2.097
12	0.1185	1.532	1.960
13	0.1041	1.534	1.929
14	0.1692	1.524	2.028
15	0.2204	1.583	2.105
16	0.2191	1.651	2.131
17	0.2055	1.961	2.226
18	0.1424	2.207	2.207
19	2.7079	2.150	3.047

Notes: All quantities in residual units, from the estimated model. $\hat{v}_b^*$ is the nomination threshold solved from $h_{1,b}$; $\hat{c}_b^{*L}$ and $\hat{c}_b^{*H}$ are the early-learner and later-learner (non-nominator) cost cutoffs from $h_{2,b}$ and $h_{3,b}$. All 19 bins are reported.

Table 19: Moment Fit

Moment	Description	Data	Model
m_1	Mean trigger rate $\Pr(M \geq 1)$	0.00664	0.00325
m_2	ZTP mean nominations $\mathbb{E}[M \mid M \geq 1]$	1.056	1.002
m_3	ZTP variance of nominations $\text{Var}[M \mid M \geq 1]$	0.0784	0.00163
m_4	Mean total bidders $\mathbb{E}[I \mid M \geq 1]$	3.318	3.240

Notes: The four active GMM moments (Appendix B.5), each reported as the equal-weight average across all 19 bins of s_t ; “Data” are the empirical bin moments and “Model” the fitted values at the estimated parameters (Table 2). m_1 is matched in logs and m_4 carries weight 1.6; m_2 and m_3 carry unit weight. Every bin is matched, including bin 19 (two triggered auctions), whose conditional moments are defined. Figure 12 (Appendix C.2) shows the fit bin by bin.

B.10 Value Functions and Equilibrium Cutoffs

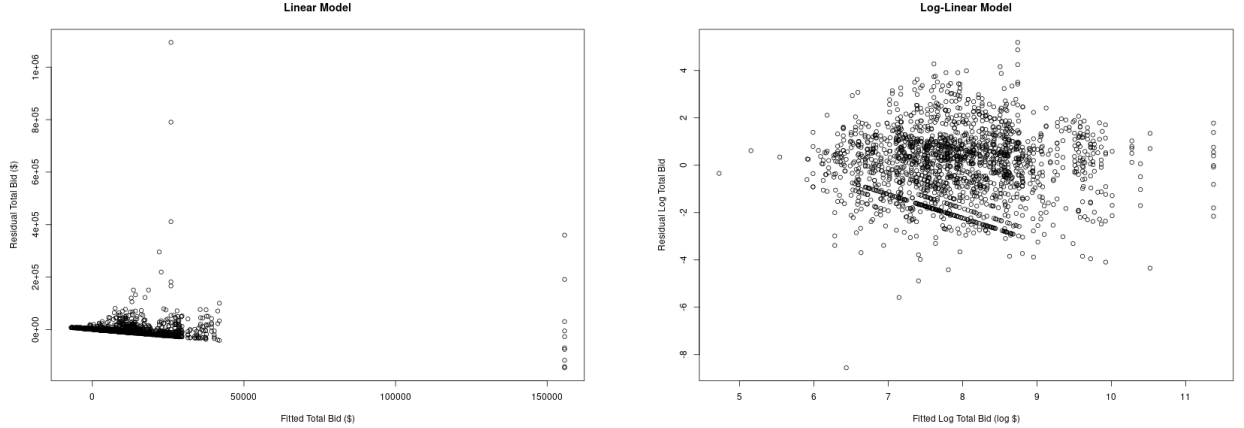
Figure 13 presents the estimated value functions and cost distribution for three representative bins: Bin 1 (the most promising tracts), Bin 10 (the median bin), and Bin 19 (the least promising tracts). The left panels show the estimated value functions $U_{l=1}(c)$ and $U_{l=2}(c)$. Both are linear and decreasing in learning cost c , but their slopes differ sharply. The early learner value function has slope -1 : every unit increase in learning cost reduces the expected payoff one-for-one, because learning is paid upfront and unconditionally. The non-nominator value function is nearly flat (slope ≈ -0.002) function is nearly flat (slope $-\Pr(M \geq 1)$, about -0.004 in Bin 10): a non-nominator pays learning cost only if the auction is triggered in Stage 2, so the expected cost is discounted by the small equilibrium trigger probability $\Pr(M \geq 1)$.

The two functions cross at the early learner cutoff c_L^* (in residual units); above c_L^* , late-learning dominates early learning, and the non-nominator value crosses zero at c_H^* , beyond which exit is optimal. Converted to dollars through the bin's multiplier ($c^* \cdot s_b$), both thresholds lie in the right tail of the shared cost distribution, whose parameters $(\hat{\xi}_c, \hat{\omega}_c, \hat{\alpha}_c) = (1,712, 690, -126.9)$ imply a mean of $\approx \$1,161$ and standard deviation $\approx \$416$. The right panels show the consequence. In Bins 1 and 10 the dollar cutoffs lie beyond the support of the cost distribution, so essentially all arriving bidders learn early; this is representative of the highly promising bins. In Bin 19 (lowest s , least promising) the dollar cutoffs fall into the body of the cost distribution and cost does screen participation (the early learning share falls to $\approx 3.5\%$; see Section 6). Even where cost imposes no screen, however, the low nomination rate observed in the data is driven by the low pre-period arrival rate $\hat{\lambda}_{\text{pre}} \approx 0.005$, which makes early (learning) arrivals rare even though exploration is individually worthwhile.

B.11 Residual Diagnostics for Bid Homogenization

Figure 9 compares residual-versus-fitted plots for two specifications of the bid-homogenization regression.

The formal Breusch-Pagan / White heteroskedasticity logic is as follows. Under a linear specification $b_{istc} = \exp(Z_i) \cdot v_{istc}$, the residual variance grows with $\exp(2Z_i)$, which depends on the fitted value; OLS is inefficient and standard errors are invalid. Under the log-linear specification, $\log b_{istc} = Z_i + \log v_{istc}$, the residual $\hat{\varepsilon}_{istc} = \log v_{istc} - \mathbb{E}[\log v_{istc} | X_t]$ has approximately constant variance. The diagnostic plots in Figure 9 confirm this pattern in the data.



(a) Linear Model: residual total bid (\$) vs. fitted total bid (\$).

(b) Log-Linear Model: residual log total bid vs. fitted log total bid.

Figure 9: Residual Diagnostic Plots for the Bid-Homogenization Regression.

Notes: Panel (a) plots residuals from a linear regression of total bid (per-acre bid $\times$ acreage, in dollars) on season fixed effects, cluster fixed effects, royalty rate, distance to nearest well, tract size, and spud-date age. The residuals fan out dramatically with fitted values, indicating severe multiplicative heteroskedasticity. Panel (b) plots residuals from the same specification with the log of total bid as the dependent variable (equation 29 in the text). Variance is approximately constant across the support, confirming the appropriateness of the multiplicative (log-linear) specification.

B.12 Geographic Clustering

Using location data, I define a cluster to group auctions that are geographically close to one another, in order to control for geological similarity via cluster fixed effects in the bid-homogenization regression (Appendix B.1). Tracts that are closely located are more likely to share similar geological characteristics—such as subsurface composition and basin correlation—each of which affects learning costs and development revenue.

I implement a clustering algorithm using a 7.5-mile proximity threshold, following [Hendricks et al. \[2003\]](#) and [Nguyen \[2022\]](#)’s approaches to defining *neighboring tracts*.⁵⁶

Definition 3 (Geographic Clusters). *Two mineral rights auctions (tracts) X_1 and X_2 belong to the same cluster if there exists a sequence of auctions $\{y_1, y_2, \dots, y_k\}$ such that the geographic distance between centroids of consecutive tracts y_i and y_{i+1} (for $i = 1, \dots, k - 1$), and the distances between centroids of y_1 and X_1 and between centroids of y_k and X_2 , are all at most 7.5 miles.*

⁵⁶Both [Hendricks et al. \[2003\]](#) and [Nguyen \[2022\]](#) define neighboring tracts using latitude and longitude thresholds (0.11 degrees of latitude and 0.12 degrees of longitude). I convert these to miles to provide more intuitive distance measures for the 7.5-mile threshold used here.

The algorithm computes Haversine distances between all tract centroids, initially assigns each tract to its own cluster, then iteratively merges clusters whenever any two tracts fall within the 7.5-mile threshold. This ensures transitivity: any two tracts in the same cluster are connected through a chain of neighbors each separated by at most 7.5 miles. Time is not considered—tracts from different auction seasons may belong to the same cluster if they are geographically proximate. The algorithm yields 36 clusters in total, ranging from single isolated tracts to dense concentrations of up to 179 tracts, with an average of 24.4 tracts per cluster.

B.13 Value of Nomination: Simulation Details

For each bin b I draw a single shared pool of $N_{\text{pool}} = 200,000$ candidate auctions and evaluate all five worlds on the *same* pool with common random numbers, so world-to-world differences are not contaminated by sampling noise. Each candidate auction t is built as follows.

- **Common component.** Draw one auction-specific factor W_t . Every bidder’s value in auction t is generated through the estimated Gaussian equicorrelation copula ($\hat{\rho} = 0.8605$), $v = F_V^{-1}(\Phi(\sqrt{\rho} W_t + \sqrt{1-\rho} \varepsilon))$, so values within an auction are correlated while W_t is independent across auctions. A fresh W_t is drawn for each of the 200,000 auctions.
- **Early (potential nominator).** One early value e_t off W_t ; the tract is “nominated” if $e_t > v^*$. Worlds $\mathcal{W}_1, \mathcal{W}_2$ keep the nominated subset; $\mathcal{W}_3$ drops $e_t > v^*$ and keeps the tracts with $e_t \geq r$; $\mathcal{W}_4$ uses the whole pool as arbitrary tracts.
- **Later arrivals.** A zero-truncated Poisson(λ_{post}) number of later bidders, each with a value off the same W_t and an independent learn draw.

The worlds differ only in three toggles—(i) whether later bidders price off the signal (F_{post}) or the prior (F_V); (ii) which tracts are drawn—those with $e_t > v^*$, those with $e_t \geq r$, or all of them; (iii) whether the early firm is a guaranteed bidder or arrives, learns, and bids only by chance—and in the arrival rate (the no-nomination world $\mathcal{W}_4$ uses $\lambda_{\text{pre}} + \lambda_{\text{post}}$). In $\mathcal{W}_1$ – $\mathcal{W}_3$ the early firm is present on every tract and always bids; in $\mathcal{W}_4$ it is present with probability $1 - e^{-\lambda_{\text{pre}}}$, learns only if its cost is below the cutoff, and bids only if its value clears the reserve. Participation uses re-solved cost cutoffs under the world’s information: the learn-vs-exit cutoff $\bar{c}_{\text{sig}}$ under the posterior and $\bar{c}_{\text{nos}} = \bar{c}_{\text{sig}} - \Delta$ under the prior, with Δ the posterior-minus-prior expected surplus from the analytic copula. A bidder is *active* (counts, and can win) only if it both learns and clears the reserve $v \geq r$.

Each auction is resolved as a second-price sale conditional on at least one active bidder: revenue is the second-highest active value (or the reserve with a single active bidder) and the winning bid is the highest active value. I report outcomes per sold auction and the sale rate separately.

Standard errors are from a two-phase bootstrap with $R = 2,000$ replications. Phase 1 computes each pool auction's outcome in each world once. Phase 2 resamples a fixed 800 auction *indices* per bin into those precomputed outcomes and re-averages; no re-simulation is needed, so large R is cheap. The reported s.e. is the standard deviation of the resampled effect across replications, and the pooled figures are tract-count-weighted across bins. The common resample size (800 per bin, rather than each bin's handful of observed auctions) puts every bin's sampling uncertainty on a comparable footing, so the per-bin decomposition is informative and not swamped by the smallest bins; it should be read as the precision one *would* attain with 800 auctions per bin, not the raw data count.

C Estimation and Counterfactual Figures and Tables

This appendix collects supplementary material for the Estimation Results (Section 6) and Counterfactual Analysis (Section 7): robustness of the attention function, the per-bin attention-implied arrival rates used in the attention counterfactual, the nominator’s surplus by own value and the nomination threshold, per-bin welfare results, and the measurement of wasteful learning.

C.1 Attention Function: Robustness

Figure 10 reproduces the bidding-stage panel of Figure 5(b) after dividing each cell’s bids by $\Pr(v > v_b^*)$, the model-implied probability that a draw clears the bin’s nomination threshold; this nets out the mechanical tendency of low- v^* bins to attract more bidders. Re-estimating the pooled attention function with these adjusted bidding points leaves the elasticity essentially unchanged ($\hat{\kappa} \approx 1.15$, against 1.16 after the adjustment), confirming that the attention pattern is not an artifact of cross-bin differences in participation thresholds. Figure 11 reports the pooled regression of equation (28) on linear axes (the main-text Figure 6 uses log-log axes).

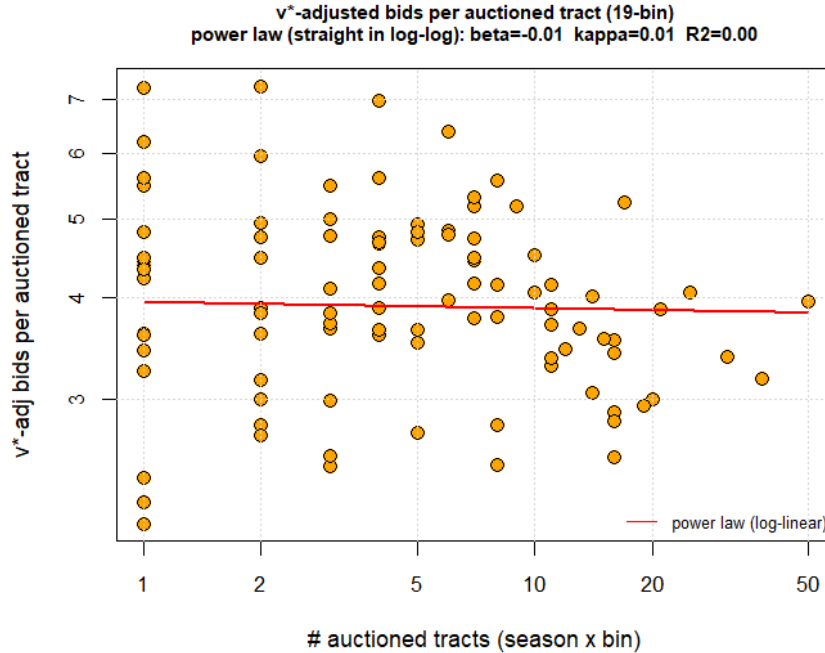


Figure 10: Bidding stage, v^* -adjusted. Bids per auctioned tract, divided by $\Pr(v > v_b^*)$, against the number of tracts auctioned (season $\times$ bin). The v^* adjustment leaves the pooled attention elasticity $\hat{\kappa} \approx 1.15$ essentially unchanged relative to the main-text fit.

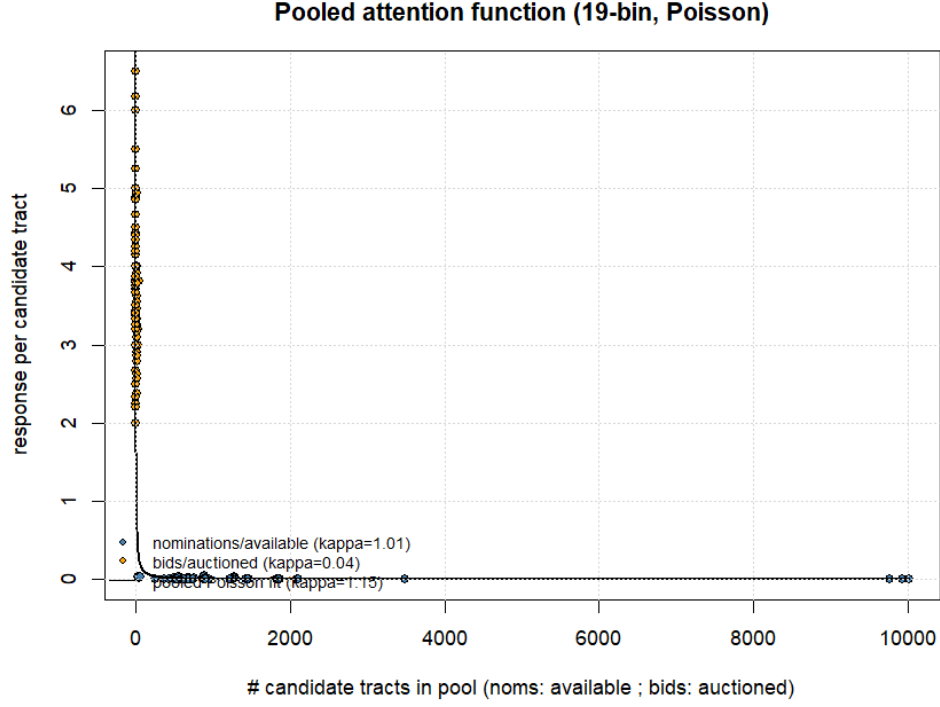


Figure 11: Pooled attention function, linear axes. Same data and pooled power-law fit as Figure 6, plotted on linear rather than log–log axes.

Attention also varies with how promising a tract is. Figure 12a (Appendix C.2) plots, for each bin, the share of available tracts that receive at least one nomination. Because almost every nominated tract receives exactly one nomination, this share is essentially nominations per available tract. It is about 1–2% in the most promising bins and falls to roughly 10^{-4} in the least promising ones. The estimated model holds λ_{pre} fixed across bins, so it can generate cross-bin differences only through cost and value screening. Those forces barely bind in the more promising bins, so the model predicts a nearly flat trigger rate through bin 12: it under-predicts for the most promising tracts and over-predicts for the least promising ones. The remaining gradient is consistent with potential bidders directing their limited attention toward higher-stakes tracts [Caplin et al., 2019]. Letting the pre-nomination arrival rate vary with the tract’s observable value multiplier s_b is a natural extension.

C.2 Per-Bin Moment Fit

Figure 12 compares each of the four estimation moments (Appendix B.5) with its fitted value, bin by bin; Table 19 reports the averages across bins. Panel (a), the trigger rate, is discussed in Appendix C.1.

Panels (b) and (c) show that nominated tracts almost always receive a single nomination.

With a small Poisson rate Λ_{nom} , the model predicts a mean of essentially one and a variance of essentially zero in every bin. The data agree wherever no tract drew a second nomination (bins 3, 10, and 13–19). In the other bins the mean is 1.04–1.27 and the variance 0.04–0.50, outside the model’s intervals. This is the overdispersion in nomination counts discussed in Appendix B.7.

Panel (d) shows that the model fits the number of bidders well in the more promising bins: in bins 1–14 the data lie inside the model’s interval in every bin except bin 6. In the least promising bins the model predicts fewer bidders—down to about one in bin 19—because the learning cost screens out most arrivals there (Figure 13). The data stay between 2.0 and 3.9 bidders and lie above the model’s interval in bins 15, 17, 18, and 19, but each of these bins contains only 2 to 10 auctions.

C.3 Per-Bin Fitted Arrival Rate from the Attention Function

This appendix derives the per-bin fitted nomination arrival rates $\hat{\lambda}_b$, which read the estimated attention function off at each bin’s available-tract pool.

Attention function. The attention function estimated in Section 6 (equation (28); pooled $\hat{A} \approx 23.9$, $\hat{\kappa} \approx 1.15$) gives the expected response per candidate tract, $r(n)$, as a function of the pool size n . Here I read it off at each bin’s available-tract pool to obtain the per-bin fitted nomination arrival rate.

Nomination response. For an available tract the response is the *total number of nominations it receives*, so in bin b , season s ,

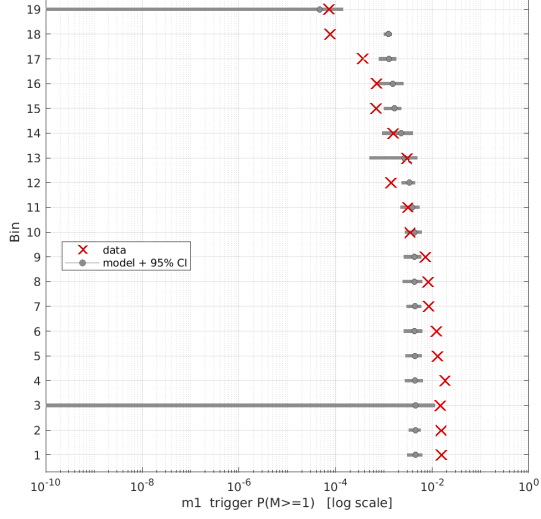
$$r_{bs}^{\text{nom}} = \frac{\sum_{t \in b,s} M_t}{N_{bs}^{\text{avail}}}, \quad (66)$$

where M_t is tract t ’s nomination count ($M_t = 0$ for the vast majority of available tracts) and N_{bs}^{avail} is the number of available tracts. Equivalently $r_{bs}^{\text{nom}} = m_{bs}^{(1)} m_{bs}^{(2)}$, the product of the trigger rate $m_{bs}^{(1)} = \Pr(M \geq 1)$ and the mean nominations per nominated tract $m_{bs}^{(2)} = \mathbb{E}[M \mid M \geq 1]$.

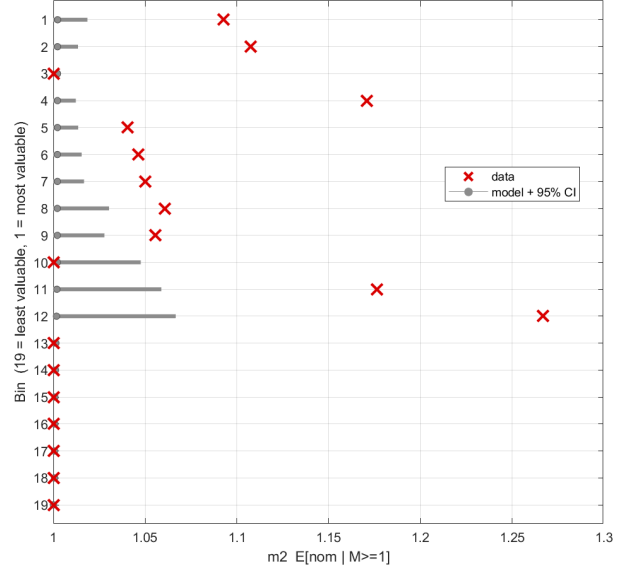
Bin pool size. For each bin b define the representative available-tract pool as the median, across the eight seasons, of the number of available tracts in that bin,

$$\bar{n}_b = \text{median}_s N_{bs}^{\text{avail}}. \quad (67)$$

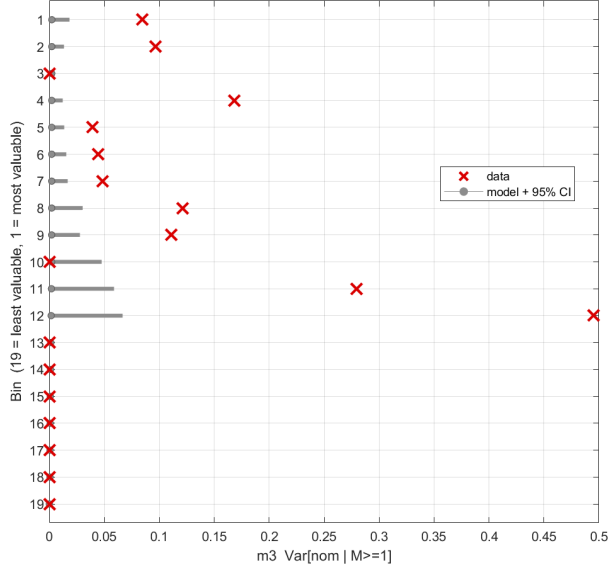
basic_untrunc m1 trigger $P(M \geq 1)$: data (red x) vs model (o, gray bar = 95% CI)



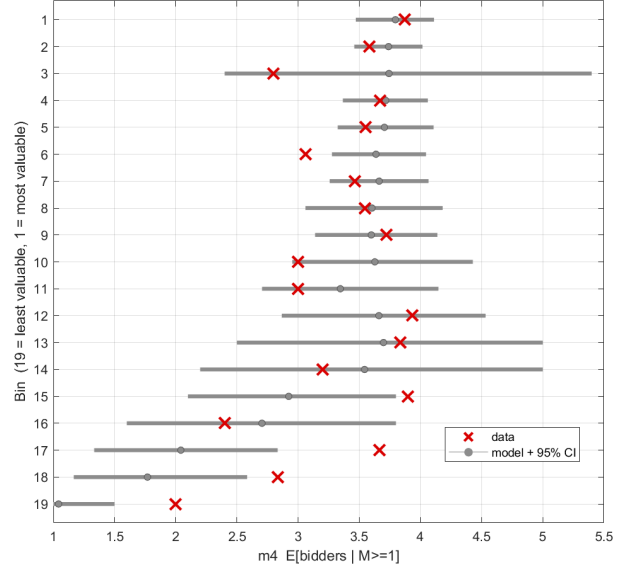
(a) Trigger rate $\Pr(M \geq 1)$, log scale



(b) Mean nominations $\mathbb{E}[M \mid M \geq 1]$



(c) Variance of nominations $\text{Var}[M \mid M \geq 1]$



(d) Mean bidders $\mathbb{E}[I \mid M \geq 1]$

Figure 12: Per-bin moment fit, data vs. model. Red crosses: data moment in each bin. Gray dots: fitted moment at the estimated parameters (Table 2); gray bars: 95% intervals from a parametric bootstrap at the estimates, using each bin's sample size. Bins run from the least promising tracts (bin 19, bottom) to the most promising (bin 1, top). In panel (a) the intervals for bins 3 and 19 include zero and therefore extend to the left edge of the log axis.

Available counts are nearly constant across seasons, so the median is a robust representative pool size; summed over bins $\sum_b \bar{n}_b \approx 28,000$, partitioning the full available-tract universe.

Fitted per-bin arrival rate. Reading the attention function (28) off at $\bar{n}_b$,

$$\hat{\lambda}_b = \hat{A} \bar{n}_b^{-\hat{\kappa}} = 23.9 \bar{n}_b^{-1.15}, \quad (68)$$

the fitted total-nominations-per-available-tract rate for a bin whose available pool is $\bar{n}_b$.

Low-attention arrival rate (aggregating the per-bin fits). The low-attention rate is the world where attention is diffused over the full $\approx 28,000$ number of tracts (n). Rather than evaluating the curve at $n = 28,000$ directly (a $\sim 3\times$ out-of-range extrapolation that overstates dilution), I aggregate the *within-range* per-bin fits—predicted nominations $\sum_b \hat{\lambda}_b \bar{n}_b$ over total available tracts $\sum_b \bar{n}_b$:

$$\hat{\lambda}_{\text{low}} = \frac{\sum_b \hat{\lambda}_b \bar{n}_b}{\sum_b \bar{n}_b} = \frac{\sum_b \hat{A} \bar{n}_b^{1-\hat{\kappa}}}{\sum_b \bar{n}_b} \approx 0.0058. \quad (69)$$

Each $\bar{n}_b$ lies inside the 50–9,900 data range, so no extrapolation is used. The aggregate ≈ 0.0058 lands essentially on the structurally estimated pre-nomination arrival rate $\hat{\lambda}_{\text{pre}} \approx 0.005$ (Table 2).

High-attention arrival rate (concentrated pool). The high-attention rate is the counterfactual in which attention is concentrated on the smaller *offered* set, as in the auction stage. I again use the pooled attention function (68) and aggregate over the per-bin *offered* pools. Writing $\bar{n}_b^{\text{off}}$ for the median number of tracts offered in bin b ,

$$\hat{\lambda}_{\text{high}} = \frac{\sum_b \hat{\lambda}_b^{\text{off}} \bar{n}_b^{\text{off}}}{\sum_b \bar{n}_b^{\text{off}}} = \frac{\sum_b \hat{A} (\bar{n}_b^{\text{off}})^{1-\hat{\kappa}}}{\sum_b \bar{n}_b^{\text{off}}} \approx 4.2, \quad \hat{\lambda}_b^{\text{off}} = \hat{A} (\bar{n}_b^{\text{off}})^{-\hat{\kappa}}, \quad (70)$$

This is of the same order as the structural post-nomination arrival rate $\hat{\lambda}_{\text{post}} \approx 2.9$ (Table 2).

The two ends of the *same* pooled attention function therefore line up with the two structural arrival rates, $\hat{\lambda}_{\text{low}} \approx 0.006$ ($\approx \hat{\lambda}_{\text{pre}}$) $\longrightarrow$ $\hat{\lambda}_{\text{high}} \approx 4.2$ ($\sim \hat{\lambda}_{\text{post}}$). These are the pre-nomination arrival inputs used in the counterfactual analysis (Section 7).

Remark (zero cells). 31% of the nomination season $\times$ bin cells receive *zero* nominations (a real response of zero per tract). The Poisson GLM keeps them; a log-log OLS of $\log(\text{rate})$ on $\log(\text{pool})$ must drop them and so understates dilution. The *pooled* $\hat{\kappa}$ is robust to this

choice of specification (1.13 OLS vs. 1.15 Poisson) because it is identified mainly by the large gap between the nomination and auction clusters.

C.4 Nominator Surplus and the Nomination Threshold

Figure 14 plots the nominator’s expected auction surplus, the left-hand side of (10), against her own value v at the estimates, for Bins 1, 10, and 19. The surplus is strictly increasing in v in every one of the 19 bins, not only the three shown, so a bidder who has learned v nominates if and only if v exceeds the value at which the curve crosses the fee. The nomination decision is therefore a threshold rule even though, with positively correlated values, a higher own value also signals stronger rivals.

The figure also shows how the fee screens tracts. Raising it from the actual \$10 to an illustrative \$100, holding rivals’ behavior at the estimates, raises the value a bidder needs before nominating from \$345 to \$1,016 in Bin 1, from \$211 to \$779 in Bin 10, and from \$331 to \$426 in Bin 19. A higher fee thus selects better tracts, at the cost of fewer nominations.

C.5 Seller-initiated Auctions With Information

C.6 Headline Totals Valuing Each Tract at Its Own Multiplier

The abstract and the introduction report the value of nomination as totals over the 728 tracts in the estimation sample, valuing each tract at its own multiplier s_t : each tract’s outcome is its bin’s simulated outcome in residual units multiplied by the tract’s s_t , and outcomes are summed over tracts. Section 7 instead values each bin at its median tract (Table 3). Table 20 reports the totals on the own-multiplier basis. The two bases give nearly the same percentage gain (44% versus 45%) and the same split across channels. The own-multiplier totals are larger in dollars because the multiplier is right-skewed within bins, so the average tract is worth more than the median tract, and because 23 tracts lie above the range of the top bin (see the table notes).

C.7 Per-Bin Welfare by Party and Concentrated Attention

Figures 16–17 report the per-bin welfare results underlying Section 7: the value-of-nomination decomposition by party and the concentrated-attention channel ($\mathcal{W}_5 \rightarrow \mathcal{W}_6$), and the by-world seller revenue and sale rate. In each figure the horizontal axis labels bins by their multiplier s_b and runs from the least promising, lowest s_b tracts (Bin 19, left) to the most promising, highest s_b (Bin 1, i.e. closest to existing wells and largest; right).

Table 20: Value of nomination over the 728 tracts, valuing each tract at its own multiplier s_t

<i>Panel A: Outcomes</i>				
	No nomination ($\mathcal{W}_4$)	Nomination ($\mathcal{W}_1$)	Difference	
Seller revenue (\$ million)	6.86	9.91	3.05 (0.57)	
Revenue per offered tract (\$)	9,418	13,607	4,189	
Revenue per sold tract (\$)	10,472	13,607	3,135	
Sale rate (%)	89.9	100.0	10.1	
Bidders per tract	2.61	3.72	1.11	

<i>Panel B: Value of nomination by party and channel (\$ million)</i>				
	Information spill over ($\mathcal{W}_1 - \mathcal{W}_2$)	More valuable tracts ($\mathcal{W}_2 - \mathcal{W}_3$)	Guaranteed bidder ($\mathcal{W}_3 - \mathcal{W}_4$)	Total (value of nom.) ($\mathcal{W}_1 - \mathcal{W}_4$)
Seller	0.00 (0.00)	0.04 (0.60)	3.01 (0.54)	3.05 (0.57)
Nominator	0.00 (0.00)	0.00 (0.35)	2.15 (0.26)	2.15 (0.24)
Non-nominator	0.00 (0.00)	0.02 (0.50)	-1.93 (0.53)	-1.91 (0.53)
Total welfare	0.00 (0.00)	0.06 (1.00)	3.22 (0.93)	3.28 (0.97)

Each tract's outcome is its bin's simulated outcome in residual units multiplied by the tract's own multiplier s_t , summed over the 728 tracts in the estimation sample; Table 3 instead values each bin at its median tract. Worlds and channels are as in Table 3, and $\mathcal{W}_4$ counts tracts that draw no bid as zero revenue. Bootstrap standard errors in parentheses, from the same 1,000 resampling draws as Table 3, rescaled tract by tract. Revenue per sold tract rises by 30%. Twenty-three of the 728 tracts have a multiplier (s_t between \$7,466 and \$9,332) above the range of the top bin; they are valued with Bin 1's outcomes, which is why the sale rate is 89.9% rather than the 89.8% in Section 7. Excluding them, seller revenue rises from \$6.28 million to \$9.10 million (\$2.82 million, 45%). Rows may not sum exactly because of rounding.

Table 21: Seller-initiated cherry-picking by observable value versus nomination

Bins offered	Top % auctioned	Top % available	Bidders per tract	Winning bid (\$)	Sale rate (%)	Revenue (\$M)
1	15.3	3.3	2.78	17,184	93.0	11.63
1-2	36.5	7.9	2.74	14,453	92.2	9.71
1-3	37.2	8.0	2.74	14,390	92.2	9.66
1-4	48.8	10.0	2.72	13,401	92.0	8.98
1-5	59.3	12.5	2.72	12,578	91.9	8.41
1-6	68.5	15.0	2.70	11,941	91.6	7.96
1-7	77.0	18.2	2.69	11,321	91.5	7.54
1-8	81.7	20.0	2.69	11,007	91.3	7.32
1-9	86.8	22.5	2.68	10,660	91.2	7.08
1-10	89.8	25.0	2.68	10,459	91.2	6.94
1-11	92.2	27.5	2.67	10,309	91.0	6.83
1-12	94.3	32.8	2.67	10,153	91.0	6.72
1-13	95.2	33.5	2.67	10,089	91.0	6.68
1-14	95.9	35.0	2.66	10,038	90.9	6.65
1-15	97.3	41.7	2.65	9,936	90.8	6.57
1-16	98.0	45.0	2.64	9,886	90.7	6.53
1-17	98.9	52.5	2.63	9,834	90.4	6.47
1-18	99.7	87.5	2.61	9,788	90.1	6.42
1-19	100.0	100.0	2.61	9,788	89.8	6.40
Nomination ($\mathcal{W}_1$)			3.72	12,743	100.0	9.28

The seller runs no nomination and offers 728 tracts arbitrarily from the listed bins, in proportion to their auction shares ($\mathcal{W}_4$ restricted to those bins); row 1-19 is the no-nomination benchmark $\mathcal{W}_4$. “Top % auctioned” is the cumulative share of auctioned tracts in the listed bins; “top % available” is the cumulative share of tracts available in 2017Q1. Bidders are per offered tract; the winning bid is per sold tract. Each bin is valued at its median tract.

C.8 Measuring Wasteful Learning off the $\mathcal{W}_1$ and $\mathcal{W}_4$ Simulations

I measure *wasteful learning* by reading it directly off the $\mathcal{W}_1$ and $\mathcal{W}_4$ simulations, rather than from a closed form. Each world is simulated as a large pool of tract draws; on every tract, post-nomination (*late*) arrivals each draw a private learning cost and, upon learning, a value. For each world $w \in \{\mathcal{W}_1, \mathcal{W}_4\}$, I index the late arrivals i in the pool and record (i) the cost draw c_i ; (ii) whether it learns, $0 < c_i < \bar{c}_w$, i.e. a positive cost below the world’s learning cutoff ($\bar{c}_{\text{sig}}$ under $\mathcal{W}_1$, i.e. $= c_H^*$ from the original estimates), which conditions on the nomination signal, and $\bar{c}_{\text{nosig}}$ under $\mathcal{W}_4$, which does not); and (iii) conditional on learning, whether it draws $v_i < r$ and so does not bid. The measure is

$$\text{WL}_w = \frac{\sum_{\text{late } i} \mathbb{I}\{0 < c_i < \bar{c}_w\} \mathbb{I}\{v_i < r\} c_i}{\sum_{\text{late } i} \mathbb{I}\{0 < c_i < \bar{c}_w\}},$$

the average learning cost sunk by a late learner who ends up unable to bid. Only post-nomination arrivals enter the sum; the early nominator is excluded, since she has $v \geq v^* \geq r$ and never wastes. Costs are clamped at $\max(0, c)$ (the skew-normal left tail may draw $c < 0$ which I drop to focus on ‘wasteful’ learning), and the denominator counts late learners with a strictly positive cost below the cutoff and therefore learned.

The two worlds differ only in the *distribution of tracts* their late learners face. In $\mathcal{W}_4$ the seller offers arbitrary tracts—including those on which the early arrival drew $v < v^*$ (a low common value) but which are auctioned regardless—so late learners frequently draw $v < r$ and waste their cost. In $\mathcal{W}_1$ late learners are on nominated tracts, where an early arrival has already cleared $v \geq v^*$; they therefore draw from the *posterior* and usually find $v \geq r$.

I find $\text{WL}_{\mathcal{W}_4} \approx \$120$ per late learner, with 10.6% drawing $v < r$ —about \$272 per auctioned tract (paid by 2.3 number of late learners per tract). The reduction nomination delivers is clearest when I split the $\mathcal{W}_4$ tracts by whether the early arrival cleared v^* . On the tracts it would not have been nominated (early learner draws $v < v^*$, those $\mathcal{W}_1$ never offers) 53.6% of late learners draw $v < r$, wasting about \$605 per learner. On the tracts it would have been nominated (early learner draws $v \geq v^*$, which $\mathcal{W}_1$ keeps) only 5.2% falls below the reserve, resulting in about \$59 wasteful learning per learner. Nomination’s screen drops exactly the high-waste tracts, which is why wasteful learning halves, from \$272 to \$140 per auctioned tract.⁵⁷

⁵⁷Because learning cost and value are drawn independently, WL_w also equals the closed form $\Pr(v < r \mid w) \cdot \mathbb{E}[c \mid 0 < c < \bar{c}_w]$ —the learning a late learner sinks, times the probability it is wasted. The simulation read-off and this closed form agree to Monte-Carlo precision (within about 0.4%). The split above holds the learning cutoff at the no-signal level, isolating the value-distribution (posterior) channel; allowing the

C.9 Bid Timing and Revisions

This appendix documents when bidders bid and how often they revise, which Section 2 uses to justify treating the auction as a second-price sealed-bid auction. Figure 19 shows when bids are placed relative to the closing time, Figure 20 how often bidders revise, Figure 21 how revision varies with when a bidder first bids, and Table 22 how many last-minute bids answer a rival’s bid within seconds. A bidder–tract pair is a bidder bidding on one tract; each recorded bid has its own placement time, and revisions are the bids after a bidder’s first bid on the tract.

Table 22: Last-Minute Bids That Answer a Rival’s Bid

	Higher by \$1	Higher by more than \$1	Total
Condition A	46	95	141
Condition A + bidder had already bid	16	20	36
All bids placed with at most one minute left			1,410

Notes: Bids placed with at most one minute left before the close. Condition A: the previous bid on the tract is by a different bidder, the bid is placed within five seconds after it, and it is higher than that bid. The second row adds that the bidder had already bid on the tract. Columns split the bids by how much they exceed the previous bid.

D Theory and Proofs

D.1 Positive Correlation Details

Formal definition. Under affiliated private values (APV), the joint density f of H_V satisfies

$$f(\mathbf{v} \vee \mathbf{v}') f(\mathbf{v} \wedge \mathbf{v}') \geq f(\mathbf{v}) f(\mathbf{v}') \quad \text{for all valuation vectors } \mathbf{v}, \mathbf{v}',$$

where $\vee$ and $\wedge$ denote coordinate-wise maximum and minimum. The parametric form of H_V adopted in Section 5 generates joint distributions satisfying this property [Nelsen, 2006, Müller and Scarsini, 2005], ensuring the empirical findings are consistent with the theoretical framework underlying equilibrium existence and revenue ranking results in auction theory [Castro, 2007].

with-signal cutoff leaves the \$272 $\rightarrow$ \$140 conclusion unchanged.

Posterior stochastic dominance. The stochastic dominance result in equation (2) is a consequence of this affiliation property: for affiliated random variables,

$$\Pr(V_i > v \mid V_j \geq v^*, j = 1, \dots, m) \geq \Pr(V_i > v) \quad \text{for all } v$$

[Milgrom and Weber, 1982]. This holds for *any* H_V satisfying **positive correlation**, not only for the Gaussian copula parameterization adopted in Section 5. Therefore, the equilibrium characterization in Section 4.4 does not rely on the specific copula choice.

D.2 Robustness to Noisy Nomination

The main text models nomination as a deterministic threshold rule: $n(v) = \mathbb{I}\{v \geq v^*\}$. Here I show the framework is robust to mixed or noisy nomination strategies.

Suppose a positive fraction $p \in (0, 1)$ of nominations are uninformative: a bidder with realized value below v^* randomizes and nominates with probability p , independent of her realized value (a bidder above v^* still nominates with probability 1, as in the main text). This dilutes the informational content of a nomination. The non-nominator's effective screening probability – the probability that a nominator's value actually clears v^* , given that she nominated – becomes

$$\pi \equiv \Pr(v \geq v^* \mid \text{nominate}) = \frac{1 - F_V(v^*)}{1 - F_V(v^*) + p F_V(v^*)},$$

rather than $\pi = 1$ under the pure-strategy rule of the main text.

D.3 Posterior Updating of the Non-Nominator's Value Distribution

This section derives the mechanism through which observing m nominators above threshold v^* shifts the non-nominator's own value distribution upward, as claimed in Section 4.3.1. The mechanism operates through the one-factor representation of the Gaussian equicorrelation copula. Each normal score $Z_i = \Phi^{-1}(U_i)$ decomposes as

$$Z_i = \sqrt{\rho} W + \sqrt{1 - \rho} \varepsilon_i, \quad W, \varepsilon_i \stackrel{\text{iid}}{\sim} \mathcal{N}(0, 1),$$

where W is a shared common factor and ε_i is idiosyncratic. Conditional on $W = w$, the Z_i are independent with $Z_i \mid W = w \sim \mathcal{N}(\sqrt{\rho} w, 1 - \rho)$, so each nominator's threshold-crossing probability is $P(Z_i > z^* \mid W = w) = 1 - \Phi\left(\frac{z^* - \sqrt{\rho} w}{\sqrt{1 - \rho}}\right)$, where $z^* = \Phi^{-1}(u^*)$. Observing m

threshold-crossings updates the posterior on W via Bayes' theorem:

$$p(w \mid m \text{ crossings}) \propto \phi(w) \cdot \left[1 - \Phi\left(\frac{z^* - \sqrt{\rho}w}{\sqrt{1-\rho}}\right) \right]^m.$$

This posterior shifts rightward as m increases — each additional nominator above z^* is independent evidence that W is high. Since the non-nominator's own normal score satisfies $Z_0 = \sqrt{\rho}W + \sqrt{1-\rho}\varepsilon_0$, the rightward shift in $p(w \mid m)$ stochastically increases Z_0 , and therefore $U_0 = \Phi(Z_0)$ and the non-nominator's value $V_0 = F_V^{-1}(U_0)$.

D.4 Corner Equilibrium Cases

This appendix presents the two corner-solution equilibrium cases that arise in the nomination game when the interior-solution condition fails. Recall the definitions (equations 12–13):

$$\begin{aligned} \Delta_0 &= A - \phi[1 - F_V(v^*)] - B \quad (\text{value difference at zero cost}) \\ \Delta_{l=2} &= U_{l=1}(c_H^*) \quad (\text{value of early learning at the non-nominator cutoff}) \end{aligned}$$

where $A = \sum_{m=K-1}^{\infty} \Pr(M_{-i}=m) S_{l=1}(m)$ is the expected gross auction surplus from early learning and $B = \Pr(M \geq K) \mathbb{E}[S_{l=2}(m) \mid M \geq K]$ is the expected gross surplus from late-learning.

Case 2: Corner Solution (All Wait and Learn Later)

Condition: $\Delta_0 < 0$ and $\Delta_{l=2} < 0$.

When $\Delta_0 < 0$, the value of early learning at zero cost falls below the value of late-learning: $U_{l=1}(0) < U_{l=2}(0)$. Combined with $\Delta_{l=2} < 0$ (late-learning is still preferable even at the non-nominator cutoff), the function $U_{l=1}(c_i) - U_{l=2}(c_i)$ is strictly negative for all $c_i \in [0, c_H^*]$. There is therefore no interior crossing point, and no bidder finds it optimal to nominate.

Formally, $\pi_0^{l=1} = 0$ and the equilibrium degenerates to a mass of non-nominators (those with $c_i \leq c_H^*$) and exits. This outcome is a *coordination failure*: each potential nominator would prefer to wait for someone else to incur the nomination cost and fee, and in equilibrium no one does.⁵⁸

As shown in the center panel of Figure 4, the $U_{l=1}$ intercept lies below the $U_{l=2}$ intercept ($A - \phi[1 - F_V(v^*)] < B$), so $U_{l=2}(c_i) > U_{l=1}(c_i)$ for all c_i in the relevant range.

⁵⁸The pivotal probability $\Pr(M_{-i} = K - 1)$ is the probability that a given bidder's nomination is decisive for triggering the auction. When this probability is low (many other nominators expected) or the fee ϕ is high relative to the nomination benefit, $\Delta_0 < 0$ and the coordination failure arises.

Case 3: Corner Solution (All Learn Early)

Condition: $\Delta_{l=2} \geq 0$.

When $\Delta_{l=2} = U_{l=1}(c_H^*) \geq 0$, the value of early learning remains non-negative at the non-nominator cutoff c_H^* . Since $U_{l=1}$ is weakly decreasing in c_i and $U_{l=2}(c_H^*) = 0$ by definition of c_H^* , this means $U_{l=1}(c_i) \geq U_{l=2}(c_i)$ for all $c_i \in [0, c_H^*]$. No bidder prefers late-learning over early learning within the participation range.

In this case the equilibrium threshold is defined by $U_{l=1}(c_L^*) = 0$ (where the early learning value just breaks even, not where it crosses the late-learning value). Any bidder with $c_i < c_L^*$ learns early; any bidder with $c_i > c_L^*$ exits. The late-learning strategy is dominated and $\pi_0^{l=2} = 0$.

The intuition is that either the nomination fee is negligible relative to the expected auction surplus (low ϕ) or the pivotal probability is very high, making the nomination benefit large enough that all participating bidders prefer to bear the cost and nominate outright. As shown in the right panel of Figure 4, the $U_{l=1}$ line lies above $U_{l=2}$ everywhere in the feasible cost range.

Empirical relevance. In the ND data, both nominators and non-nominators are observed in every auction season, consistent with the Case 1 interior solution. Cases 2 and 3 are not observed in the data. They mark the limits of the mechanism, however, where a sufficiently large increase in the nomination fee ϕ would push the equilibrium from Case 1 into Case 2, eliminating nominations entirely.

D.5 Proofs for Section 4.4

D.6 Proof of Lemma 1: Poisson Arrivals of Nominators

Proof. Let $N \sim \text{Poisson}(\Lambda)$ denote the total number of arriving bidders, where $\Lambda = \lambda_{pre}$. Each arriving bidder independently becomes a competing nominator with probability $p \equiv \pi_0^{l=1} \cdot [1 - F_V(v^*)]$, reflecting two independent thinning events: the bidder chooses the early learning ($l = 1$) strategy with probability $\pi_0^{l=1}$, and conditional on that, has a value exceeding the threshold v^* with probability $[1 - F_V(v^*)]$.

I derive the distribution of M_{-i} , the number of competing nominators excluding bidder

i , directly. Conditioning on the total number of arrivals $N = j$ gives:

$$\Pr(M_{-i} = m) = \sum_{j=m}^{\infty} \Pr(M_{-i} = m \mid N = j) \Pr(N = j) \quad (71)$$

$$= \sum_{j=m}^{\infty} \binom{j}{m} p^m (1-p)^{j-m} \cdot \frac{e^{-\Lambda} \Lambda^j}{j!} \quad (72)$$

where the binomial factor $\binom{j}{m} p^m (1-p)^{j-m}$ is the probability that exactly m of the j arrivals become competing nominators. Rearranging terms,

$$\Pr(M_{-i} = m) = \sum_{j=m}^{\infty} \frac{j!}{m! (j-m)!} \cdot p^m (1-p)^{j-m} \cdot \frac{e^{-\Lambda} \Lambda^j}{j!} \quad (73)$$

$$= \frac{e^{-\Lambda} p^m}{m!} \sum_{j=m}^{\infty} \frac{\Lambda^j (1-p)^{j-m}}{(j-m)!} \quad (74)$$

Substituting $k = j - m$,

$$\Pr(M_{-i} = m) = \frac{e^{-\Lambda} p^m \Lambda^m}{m!} \sum_{k=0}^{\infty} \frac{[\Lambda(1-p)]^k}{k!} \quad (75)$$

$$= \frac{e^{-\Lambda} (\Lambda p)^m}{m!} \cdot e^{\Lambda(1-p)} \quad (76)$$

$$= \frac{e^{-\Lambda p} (\Lambda p)^m}{m!} \quad (77)$$

where the Taylor expansion $\sum_{k=0}^{\infty} \frac{x^k}{k!} = e^x$ is used and combines $e^{-\Lambda} \cdot e^{\Lambda(1-p)} = e^{-\Lambda p}$. This is the PMF of $\text{Poisson}(\Lambda p)$, so

$$M_{-i} \sim \text{Poisson}(\Lambda p) = \text{Poisson}(\lambda_{pre} \cdot \pi_0^{l=1} \cdot [1 - F_V(v^*)]) \quad \square$$

$\square$

D.7 Proof of Proposition 1: Single Crossing

Proof. Both value functions are linear in c_i :

$$U_{l=1}(c_i) = A - \phi[1 - F_V(v^*)] - c_i \quad \text{with slope } -1 \quad (78)$$

$$U_{l=2}(c_i) = B - \Pr(M \geq K) \cdot c_i \quad \text{with slope } -\Pr(M \geq K) \quad (79)$$

where $A = \sum_{m=K-1}^{\infty} \Pr(M_{-i} = m) S_{l=1}(m)$ is the expected gross auction surplus for an early learner, $B = \Pr(M \geq K) \cdot \mathbb{E}[S_{l=2}(m) | M \geq K]$ is the expected gross surplus for a non-nominator, and $\phi[1 - F_V(v^*)]$ is the expected nomination fee paid with probability $1 - F_V(v^*)$.

Since $\Pr(M \geq K) < 1$,

$$|dU_{l=1}/dc_i| = 1 > \Pr(M \geq K) = |dU_{l=2}/dc_i|. \quad (80)$$

Two lines with different slopes can intersect at most once. If they intersect at c_L^* , then

$$\begin{cases} U_{l=1}(c_i) > U_{l=2}(c_i) & \text{(learn early) if } c_i < c_L^* \\ U_{l=1}(c_i) < U_{l=2}(c_i) & \text{(learn later) if } c_i > c_L^* \end{cases}$$

□

D.8 Joint GMM Estimation of F_V and ρ

This subsection restates the Joint GMM Strategy of Appendix B.3.2, the main estimator of the copula parameter and the marginal value distribution, and reports Monte Carlo evidence on its performance. The objective is

$$(\hat{\beta}, \hat{\rho}) = \arg \min_{\beta, \rho} \sum_{i \in \mathcal{G}} \left[G_k(B(x_i; \beta); k_i, I_i, \rho) - \hat{U}_{(k_i)}^{(r_i)} \right]^2, \quad (81)$$

where $B(x; \beta)$ is a monotone B-spline (10 internal knots, degree 3) with coefficient vector β constrained to be non-decreasing, G_k is the copula order-statistic CDF (Equation 41), $\hat{U}$ are Weibull empirical ranks, and $\mathcal{G}$ is a 200-point evenly-spaced x-grid selected from the training bids. The B-spline basis is built on all training bids; only the SSR evaluation is restricted to the 200-point grid to reduce computation.

After minimising (81), the lognormal parameters $(\hat{\mu}, \hat{\sigma})$ are initialized by NLS: fitting $\Phi((\log x - \mu)/\sigma)$ to the spline values $B(x_i; \hat{\beta})$ for all training bids x_i . This provides a smooth, parametric extrapolation of F_V below v^* , but is then refined by the truncation-corrected lognormal MLE described in Appendix B.3 (Eq. 45); the MLE pair $(\hat{\mu}, \hat{\sigma}) = (0.032, 1.614)$ is the one used downstream.

Estimation uses 100 random 90/10 train/test splits of the 728 auctions. The test-set RMSE for each split is computed by evaluating the copula diagonal $D(\hat{F}_V(x_{\max}); I_r - 1, \hat{\rho})$ for each test-auction maximum bid $x_{\max}$ in an I_r -bidder auction, then comparing to the empirical Weibull rank of $x_{\max}$ among all observed winner bids.

Results. The full-sample estimates are reported in Table 16 and discussed in Appendix B.3.2.

Monte Carlo evidence for median aggregation. To provide evidence for using the median rather than the mean across the 100 train/test splits as the main estimate of the correlation parameter, I run a Monte Carlo study in which I simulate fresh datasets from the correctly-specified model across all three copula families and apply the same Joint GMM estimator, then compare the mean and median as aggregation strategies across replications. I document that under Gumbel and Clayton dependence structures, the distribution of the aggregated $\hat{\rho}$ is right-skewed, which makes the median a better summary of the true correlation parameter.

Design. For each of six DGP cells (Gumbel $\rho_0 \in \{1.5, 4.0\}$, Clayton $\rho_0 \in \{0.5, 3.0\}$, Gaussian $\rho_0 \in \{0.3, 0.8\}$) crossed with six lognormal marginal variants ($\mu \in \{-0.5, 0.0, 0.5\}$, $\sigma \in \{0.3, 0.7\}$), I draw $R = 100$ independent datasets for each auction-count level $S \in$

$\{50, 100, 700\}$. Each dataset consists of S auctions with $I_s \sim \text{Uniform}\{2, \dots, 11\}$ bidders and one observed bid vector per auction (mimicking the single-observation-per-auction structure of the empirical data). The Gaussian copula objective uses a 20-node Gauss-Hermite quadrature approximation to evaluate the copula diagonal. I apply the same Joint GMM procedure to each dataset and record $\hat{\rho}^{(r)}$ for $r = 1, \dots, 100$.

The key measure for comparison is the *normalised deviation* $\delta^{(r)} = (\hat{\rho}^{(r)} - \rho_0)/\rho_0$, and I compare Ok but these two different aggregation method across $r = 1, \dots, 100$ datasets: $\text{mean}(= \bar{\rho} = \frac{1}{100} \sum_r \hat{\rho}^{(r)})$ and $\text{median}(= \tilde{\rho} = \text{median}(\hat{\rho}^{(1)}, \dots, \hat{\rho}^{(100)}))$.

Note that the root mean squared error $\text{RMSE} = \sqrt{\frac{1}{100} \sum_i (\delta^{(i)})^2}$ characterizes the *full distribution* of individual estimates and is the same regardless of which aggregator is reported.

Results. Figure 22 shows the RMSE of the normalised deviation $(\hat{\rho} - \rho_0)/\rho_0$ as a function of S for each copula family and dependence level. All six DGP cells converge toward zero as S grows, confirming consistency of the Joint GMM estimator. The Gaussian high cell ($\rho_0 = 0.8$) converges fastest, while Clayton low ($\rho_0 = 0.5$) and Gaussian low ($\rho_0 = 0.3$) show the highest RMSE at $S = 700$ (25% and 38% respectively), reflecting the flat GMM objective surface at weak dependence.⁵⁹

Table 23 summarises the bias of each aggregator, pooled over the six lognormal marginal variants (600 replications per row).

Interpretation. Four findings emerge from Table 23 and Figures 22–23.

First, for Gumbel and Clayton families the median aggregator is consistently less biased than the mean at every sample size. At $S = 50$, the mean–median gap reaches 35 percentage points for Gumbel high dependence (+109% vs. +74%) and 18 points for Clayton high (+109% vs. +91%); by $S = 700$ the gap shrinks to $\lesssim 5$ points as the objective surface concentrates and optimizer outliers vanish. This pattern is the hallmark of a right-skewed estimator distribution driven by occasional L-BFGS-B convergence to high local minima, not by systematic model misspecification.

Second, the Gaussian copula behaves differently across the two dependence regimes. For strong dependence ($\rho_0 = 0.8$), the estimator is remarkably well-concentrated even at $S = 50$: mean and median bias are both around +16% with RMSE of 16%. By $S = 700$, bias falls to +3% and RMSE to 4% — the tightest convergence of any copula family. The bounded parameter space $\rho \in (-1, 1)$ prevents the runaway outliers seen in Gumbel and Clayton, so

⁵⁹For all three copula families, the copula diagonal evaluates to the independence case $C_r(u, \dots, u) = u^r$ as the dependence parameter approaches its independence limit. Since the moment conditions $G_k(u; I, \rho)$ are built from sums of copula diagonals, it follows that $\partial G_k / \partial \rho \rightarrow 0$ as dependence weakens: the moment conditions stop responding to ρ near independence, making the GMM objective surface flat in the ρ direction.

Table 23: Monte Carlo bias of mean and median $\hat{\rho}$ aggregators under the Joint GMM estimator, pooled over six lognormal marginal variants (600 replications per row). Bias is $(\hat{\rho} - \rho_0)/\rho_0 \times 100\%$. RMSE is the root mean squared normalised deviation of the *individual* $\hat{\rho}$ draws — it is the same regardless of which aggregator is reported. Gaussian results use the vectorised 20-node Gauss-Hermite evaluator (`strategy3_sim_validation_fast.py`).

DGP	ρ_0	S	Mean bias	Median bias	RMSE
Gumbel low	1.5	50	+57%	+50%	66%
		100	+28%	+25%	32%
		700	+5%	+5%	8%
Gumbel high	4.0	50	+109%	+74%	148%
		100	+48%	+36%	75%
		700	+14%	+10%	24%
Clayton low	0.5	50	+211%	+208%	229%
		100	+112%	+105%	126%
		700	+17%	+16%	25%
Clayton high	3.0	50	+109%	+91%	134%
		100	+60%	+54%	76%
		700	+11%	+9%	20%
Gaussian low	0.3	50	+110%	+110%	114%
		100	+65%	+67%	71%
		700	+10%	+15%	38%
Gaussian high	0.8	50	+16%	+16%	16%
		100	+9%	+10%	11%
		700	+3%	+2%	4%

mean and median agree. For weak dependence ($\rho_0 = 0.3$), the Gaussian objective is flatter: RMSE remains elevated at $S = 700$ (38%), and the mean–median gap is *reversed* (mean +10%, median +15%), consistent with occasional near-boundary estimates at $\hat{\rho} \approx 0$ pulling the mean below the median rather than above it.

Third, the estimator converges to the truth as S grows across all families. By $S = 700$ (the actual sample size in this paper), Gumbel and Clayton cells show median bias of at most 16% and RMSE of at most 25%, and Gaussian high shows median bias of 2% with RMSE of 4%.

Fourth, the empirical application uses the Gaussian copula family with an estimated $\hat{\rho} = 0.861$, placing it in the “Gaussian high” regime ($\rho_0 = 0.8$). In that regime, mean and median are nearly identical at $S = 700$ and both provide low-bias estimates. The residual mean–median gap observed in the 100 train/test splits (mean 0.866, median 0.861, SD 0.096) therefore likely reflects a combination of (i) data heterogeneity across splits and (ii) a modest right-skew from splits with fewer auctions, where the $\hat{\rho}$ distribution behaves more like the small- S case. The **median** $\hat{\rho} = 0.861$ is the preferred point estimate as it is more robust to

this residual skew.

D.9 Implementation: Handling Mixed Competition

I compute $\tilde{\Pi}(v; m, \Lambda, \rho)$, where m is the deterministic component (known competitors) and Λ is the stochastic component that captures the random variable $n \sim \text{Poisson}(\Lambda)$. First, for m known competitors and n potential competition, I determine 90th percentile support ($n_{\max} = \text{poissinv}(0.90, \Lambda) + 3\sqrt{\Lambda}$) and compute Poisson probabilities as $p_n = \frac{e^{-\Lambda}\Lambda^n}{n!}$ for $n \in \{0, \dots, n_{\max}\}$. This is then normalized as $p_n \leftarrow p_n / \sum p_n$. Next, for each n , I compute surplus with $m + n$ total competitors and take expectation as $\tilde{\Pi}(v; m, \Lambda, \rho) = \sum_n p_n \cdot \text{surplus}(v, m + n, \rho)$.

Simulation of Gaussian Equicorrelation Copula

I estimate the conditional density $f_{V_{(n:n)}|V_0}(y \mid v, \hat{\rho})$ via Monte Carlo simulation from the $(n + 1)$ -dimensional Gaussian equicorrelation copula. The simulation exploits the one-factor (“compound symmetric”) structure.

Simulation algorithm.

1. **Generate common factor:** Draw $W \sim N(0, 1)$ independently for each simulation replicate. This shared factor induces equicorrelation across all dimensions.
2. **Generate idiosyncratic shocks:** Draw $\varepsilon_j \stackrel{\text{i.i.d.}}{\sim} N(0, 1)$, $j = 1, \dots, n + 1$, independent of W .
3. **Construct correlated normals:** $Z_j = \sqrt{\hat{\rho}}W + \sqrt{1 - \hat{\rho}}\varepsilon_j$. By construction, $\text{Cov}(Z_j, Z_k) = \hat{\rho}$ for $j \neq k$.
4. **Transform to uniform marginals:** $U_j = \Phi(Z_j)$, where Φ is the standard normal CDF. The vector $(U_1, \dots, U_{n+1})$ follows the Gaussian equicorrelation copula with parameter $\hat{\rho}$.
5. **Transform to bid scale:** $V_j = F_V^{-1}(U_j)$ for $j = 1$ (own value, full distribution) and $V_j = F_{V|_{\text{trunc}}}^{-1}(U_j)$ for $j = 2, \dots, n + 1$ (competitors, truncated distribution $F_V(\cdot \mid v \geq v^*)$).
6. **Record maximum competitor bid:** $V_{(n:n)} = \max\{V_2, \dots, V_{n+1}\}$.
7. **Conditional sampling:** Retain draws where the own bid $|U_1 - F_V(v)| < \varepsilon$ (adaptive tolerance $\varepsilon = 0.02$, doubled if fewer than 100 draws survive).

8. **Kernel density estimation:** Estimate $\hat{f}_{V_{(n:n)}|V_0}(y \mid v)$ from retained $\{V_{(n:n)}^{(j)}\}$ via Gaussian KDE with Silverman bandwidth. Normalize to integrate to one over $[v^*, \infty]$.

E Toy Model and Supplementary Analysis

E.1 Simplified Nomination Model: Coordination Without Late-Learners

This appendix presents a simplified version of the nomination game that abstracts from value uncertainty and strategic late-learning. The purpose is to isolate the pure coordination problem inherent in the nomination requirement K and to provide geometric intuition for equilibrium multiplicity. The full model (Section 4) adds learning costs, endogenous late learning, and Poisson arrival rates.

E.2 Simplifications Relative to Full Model

The following simplifications transform the two-stage game with early learning ($l = 1$) to nominate and late-learning into a one-shot coordination game where the only decision is whether to pay cost c_i to nominate a parcel of known value v_i .

Assumption 1: No Value Uncertainty. All bidders know their private values v_i ex ante, eliminating learning costs and the late-learning motive. This allows us to focus purely on the coordination problem inherent in the nomination threshold.

Assumption 2: Fixed Competition. The number of potential bidders I is known and fixed (not Poisson-distributed). This removes uncertainty about the number of competitors and isolates the strategic interaction in nomination decisions.

Assumption 3: Nomination Costs Only. Bidders face only nomination costs $c_i \sim \text{Gamma}(\theta_{c1}, \theta_{c2})$, which rationalize the observed low nomination rates $\frac{M_t}{I_t}$ despite the absence of value uncertainty.

These simplifications transform the model from a complex game with sequential exploration and entry decisions into a cleaner coordination game where the trade-off between auction occurrence (extensive margin) and revenue conditional on occurrence (intensive margin) is transparent.

E.3 Nomination Decision and Equilibrium

A bidder with valuation v_i and nomination cost c_i nominates if and only if

$$h(\pi_0; K, I) \cdot \Delta(v_i; I) \geq c_i \tag{82}$$

where $h(\pi_0; K, I) = \binom{I-1}{K-1} \pi_0^{K-1} (1 - \pi_0)^{I-K}$ is the probability of being pivotal and $\Delta(v_i; I) = \int_0^{v_i} F_V(y)^{I-1} dy$ is the expected surplus from a second-price auction with $I - 1$ competitors. The symmetric equilibrium nomination probability π_0^* satisfies

$$\pi_0^* = \int_{\underline{v}}^{\bar{v}} F_C(h(\pi_0^*; K, I) \cdot \Delta(v; I)) f_V(v) dv. \quad (83)$$

When $K = 1$, nominations are strategic substitutes: $h(\pi_0; 1, I) = (1 - \pi_0)^{I-1}$ is strictly decreasing, ensuring a unique equilibrium. When $K \geq 2$, nominations become strategic complements and multiple equilibria, including a coordination-failure equilibrium at $\pi_0^* = 0$, can arise (Figure 24).

E.4 Estimation of Cost Distribution

I estimate the nomination cost distribution $F_C(c; \theta_c)$ using a Gamma specification

$$F_C(c; \alpha, \beta) = \frac{\gamma(\alpha, c/\beta)}{\Gamma(\alpha)} \quad (84)$$

where $\theta_c = (\alpha, \beta)$ are shape and rate parameters. Using the observed nomination rate $\bar{\pi}_{\text{obs}}$ and the variance of nomination rates, I form two moment conditions and estimate via GMM:

$$\hat{\theta}_c = \arg \min_{\theta_c \in \Theta_c} \mathbf{g}(\theta_c)' \mathbf{W} \mathbf{g}(\theta_c). \quad (85)$$

Table 24 presents the estimates.

Table 24: Toy Model: Estimated Parameters

Parameter	Estimate	Std. Error
Shape (α)	0.82	(0.09)
Scale (β)	34.44	(3.12)
Implied Moments		
Mean cost (\$/acre)	28.09	(3.05)
Std. dev. cost (\$/acre)	31.11	(4.21)

Standard errors in parentheses, computed via bootstrap with 1,000 replications.

E.5 Counterfactual: Nomination Threshold K

Table 25 presents counterfactual results varying $K \in \{1, 2, 3\}$ with $N = 4$ bidders. In this simplified model (no late-learning), $K = 2$ produces a reduced but positive trigger rate

and doubles conditional revenue through selection. $K = 3$ produces complete coordination failure.

Table 25: Toy Model Counterfactual: Effect of Nomination Threshold

Metric	$K = 1$ (Baseline)	$K = 2$	$K = 3$
Nomination rate (π_0^*)	19.2%	15.4%	0%
Auction occurrence $\Pr(M \geq K)$	58%	24%	0%
# Bidders (if occurs)	3.56	3.56	—
Revenue occurs (\$/acre)	120	250	—
Revenue unconditional (\$/acre)	70	60	0
<i>Change vs. baseline:</i>			
Δ Revenue occurs	—	+108%	—
Δ Revenue unconditional	—	−14%	−100%

All results at mean $N = 4$ bidders. Revenue is per-acre winning bid. “—” indicates auction never occurs. Contrast with the full structural model (Section 7), where $K = 2$ already produces zero trigger rate.

E.6 Equilibrium Multiplicity Under $K \geq 2$

The hump-shaped $G(\pi)$ function when $K \geq 2$ reflects strategic complementarities: the pivotal probability initially increases with π (making nomination more attractive), peaks at $\pi \approx \frac{K-1}{I-1}$, then declines. This non-monotonicity generates multiple equilibria, with the coordination-failure equilibrium ($\pi = 0$) always present when $K \geq 2$.

E.7 Proof of Uniqueness when $K = 1$

Proposition 4 (Uniqueness for $K = 1$). *When $K = 1$, there exists at most one equilibrium nomination probability $\pi^* \in [0, 1]$.*

Proof. When $K = 1$, the pivotal probability becomes

$$h(\pi) = (1 - \pi)^{I-1} \tag{86}$$

Taking the derivative

$$h'(\pi) = -(I - 1)(1 - \pi)^{I-2} < 0 \quad \forall \pi \in [0, 1) \tag{87}$$

The operator $G(\pi, I)$ is defined as

$$G(\pi, I) = \int_{\underline{v}}^{\bar{v}} F_C((1 - \pi)^{I-1} \cdot \Delta(v; I)) f_V(v) dv \quad (88)$$

Since F_C is monotonically non-decreasing and $h(\pi)$ is strictly decreasing, $G(\pi, I)$ is strictly decreasing in π . Therefore, the fixed point equation $G(\pi^*, I) = \pi^*$ has at most one solution.

If $G(0, I) \leq 0$, then $\pi^* = 0$ is the unique equilibrium. If $G(0, I) > 0$, then by the Intermediate Value Theorem and monotonicity, there exists a unique $\pi^* \in (0, 1)$ satisfying $G(\pi^*, I) = \pi^*$. $\square$

E.8 Proof of Multiple Equilibria when $K > 1$

Proposition 5 (Multiple Equilibria for $K > 1$). *When $K > 1$, multiple equilibria may exist, including*

1. $\pi^* = 0$ (coordination failure equilibrium)
2. One or more interior equilibria $\pi^* \in (0, 1)$

Proof. For $K > 1$, the pivotal probability is

$$h(\pi) = \binom{I-1}{K-1} \pi^{K-1} (1 - \pi)^{I-K} \quad (89)$$

Step 1: Properties of $h(\pi)$.

Taking the derivative

$$h'(\pi) = \binom{I-1}{K-1} [(K-1)\pi^{K-2}(1-\pi)^{I-K} - (I-K)\pi^{K-1}(1-\pi)^{I-K-1}] \quad (90)$$

$$= \binom{I-1}{K-1} \pi^{K-2}(1-\pi)^{I-K-1} [(K-1)(1-\pi) - (I-K)\pi] \quad (91)$$

Setting $h'(\pi) = 0$

$$(K-1)(1-\pi) = (I-K)\pi \implies \pi_{l=1} = \frac{K-1}{I-1} \quad (92)$$

This is the unique interior critical point. Since $h(0) = h(1) = 0$ and $h(\pi_{l=1}) > 0$, $h(\pi)$ is single-peaked.

Step 2: Properties of $G(\pi, I)$.

Because F_C is non-decreasing and $\Delta(v; I) \geq 0$, for each v the map $\pi \mapsto F_C(h(\pi) \Delta(v; I))$ is single-peaked at the same point as $h(\pi)$; integrating over v preserves this, so

$$G(\pi, I) = \int_{\underline{v}}^{\bar{v}} F_C(h(\pi) \cdot \Delta(v; I)) f_V(v) dv \quad (93)$$

is also single-peaked at $\pi_{l=1}$, with $G(0, I) = G(1, I) = F_C(0)$.

Step 3: Equilibrium multiplicity.

The fixed point equation $G(\pi, I) = \pi$ can have multiple solutions depending on the height of the peak $G(\pi_{l=1}, I)$. If $G(\pi_{l=1}, I) < \pi_{l=1}$, only $\pi^* = 0$ exists (coordination failure). If $G(\pi_{l=1}, I) > \pi_{l=1}$, the 45-degree line intersects $G(\cdot)$ at $\pi^* = 0$ and at least one interior point. The number of interior equilibria depends on the curvature of $G(\cdot)$ near $\pi_{l=1}$

□

F Posterior Density Ratio: $f_{\text{post}}(v \mid m)/f_V(v)$

Figure 25 provides direct evidence for the nature of the Bayesian update at the estimated parameters ($\hat{\rho} = 0.861$, $v^* \approx r$, $u^* = F_V(v^*) \approx 0$). The left panel overlays the copula posterior density $f_{\text{post}}(v \mid m)$ at $m \in \{1, 2, 4, 8\}$ against the prior density truncated at v^* (black dashed, the $m = 0$ baseline). The right panel plots the pointwise ratio $f_{\text{post}}(v \mid m)/f_{\text{prior}}(v)$.

Across the mass-bearing region the ratio is indistinguishable from 1: the posterior and truncated-prior densities in the left panel are visually almost identical, so the Bayesian update leaves the conditional distribution unchanged.

This confirms that the near-zero nomination threshold $u^* \approx 0$ renders the conditioning event $\{V_j \geq v^*\}$ nearly certain: it conveys almost no information about the common factor W through the *shape* of the conditional distribution within $[v^*, \infty)$. The operative information channel at the estimated parameters is instead the *entry probability*: the mass $\int_{v^*}^{\infty} f_{\text{post}}(v \mid m) dv = \Pr(V \geq v^* \mid m)$ rises above $\Pr(V \geq v^*)$ via the strong copula correlation ($\hat{\rho} = 0.861$), even though the shape of the conditional density is unchanged.

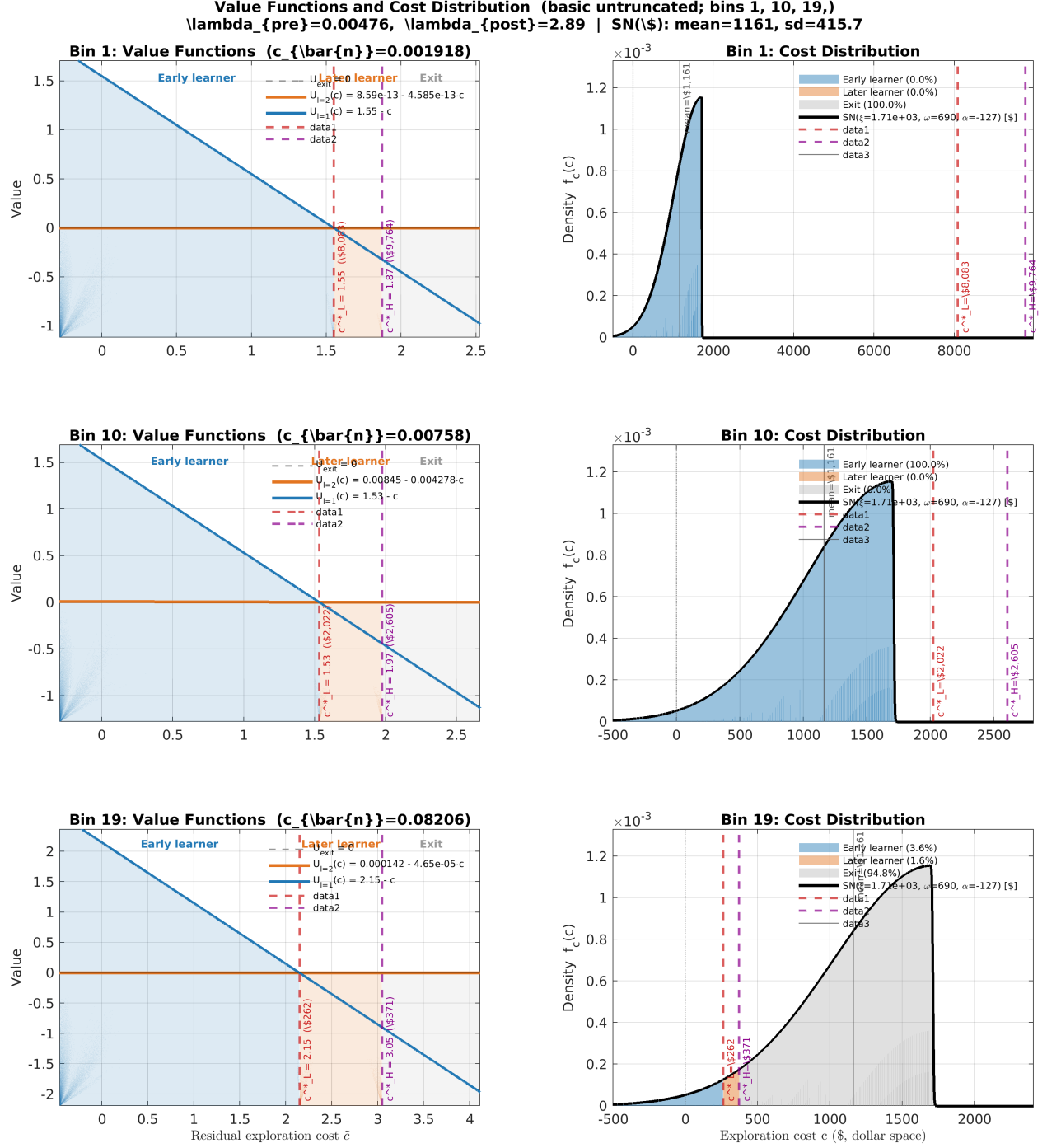


Figure 13: Value Functions and Cost Distribution (Bins 1, 10, and 19)

Notes: Left panels: early learner value $U_{l=1}(c)$ (blue) and non-nominator value $U_{l=2}(c)$ (orange) as functions of the residual learning cost $\tilde{c}$, for Bins 1, 10, and 19. Vertical dashed lines mark c_L^* and c_H^* (dollar equivalents in parentheses). Right panels: estimated Skew-Normal cost density $f_C(c)$ in dollars, with the early learner and non-nominator cutoffs marked; shaded areas give the share of arrivals choosing each action. Residual costs convert to dollars as $c = \tilde{c} \cdot s_t$, where $s_t = \exp(\hat{y}_t)$ is the tract's observable value multiplier.

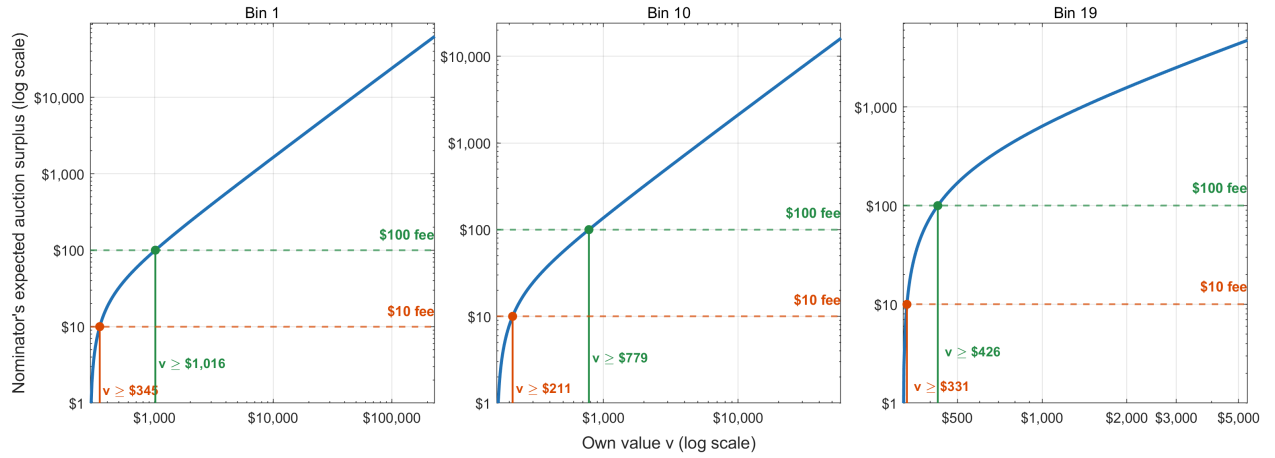


Figure 14: Nominator's Expected Auction Surplus by Own Value (Bins 1, 10, and 19)

Notes: Each panel plots the nominator's expected auction surplus, $\sum_{m \geq 0} \Pr(M_{-i} = m) \Pi(v; m, \Lambda_{nn}, H_V)$, against her own value v , in dollars and on log scales, from the reserve to the 99th percentile of F_V . Competition (Λ_{nn} and the distribution of other nominators) is held at each bin's estimated equilibrium, and the surplus is computed exactly from the Gaussian copula at $\hat{\rho} = 0.861$, with no smoothing. Dashed lines: the actual \$10 nomination fee and an illustrative \$100 fee. Vertical lines mark the value above which a bidder nominates. The \$100 thresholds are partial equilibrium: rivals' learning and nomination decisions are not re-solved.

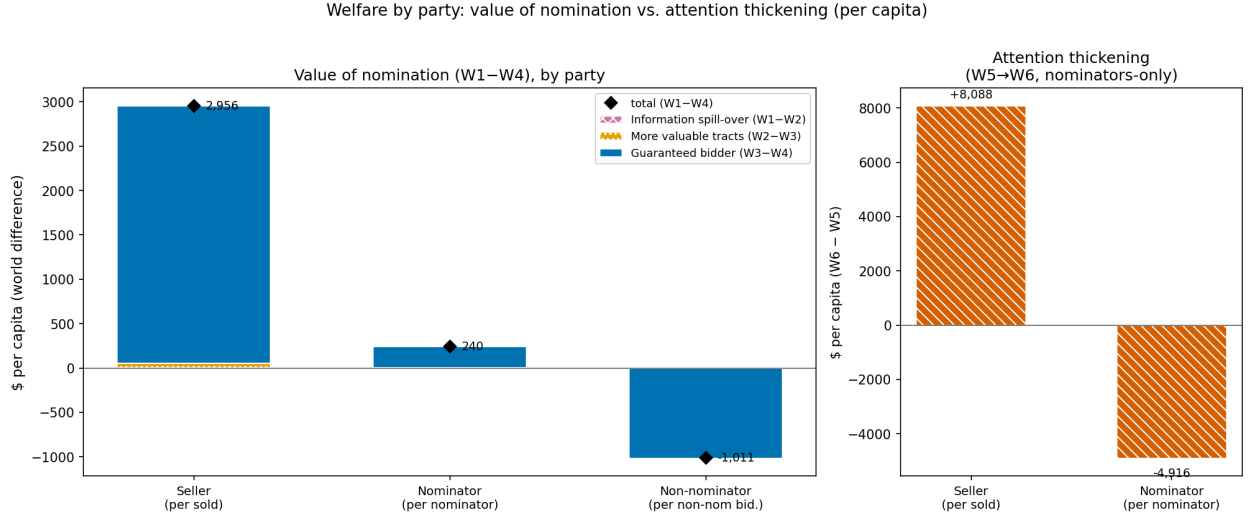


Figure 15: **Welfare by party: value of nomination vs. concentrated attention (per offered tract).** *Left:* the value of nomination ($\mathcal{W}_1 - \mathcal{W}_4$), per offered tract, for the seller, nominator, and non-nominator, decomposed into information spill over ($\mathcal{W}_1 - \mathcal{W}_2$), the selection of more valuable tracts ($\mathcal{W}_2 - \mathcal{W}_3$), and the guaranteed bidder ($\mathcal{W}_3 - \mathcal{W}_4$); the white/red diamond marks each party's total (the value of nomination). *Right:* concentrated attention ($\mathcal{W}_5 \rightarrow \mathcal{W}_6$), per tract. Per offered tract, $\mathcal{W}_4$ counts tracts that receive no bid as zero. Bars value each bin at its median tract; valuing each tract at its own multiplier s_t instead gives a seller gain of \$4,189 per offered tract (\$9,418 $\rightarrow$ \$13,607; Appendix Table 20), the basis of the abstract and the introduction. Bootstrap standard errors from the resampling bootstrap.

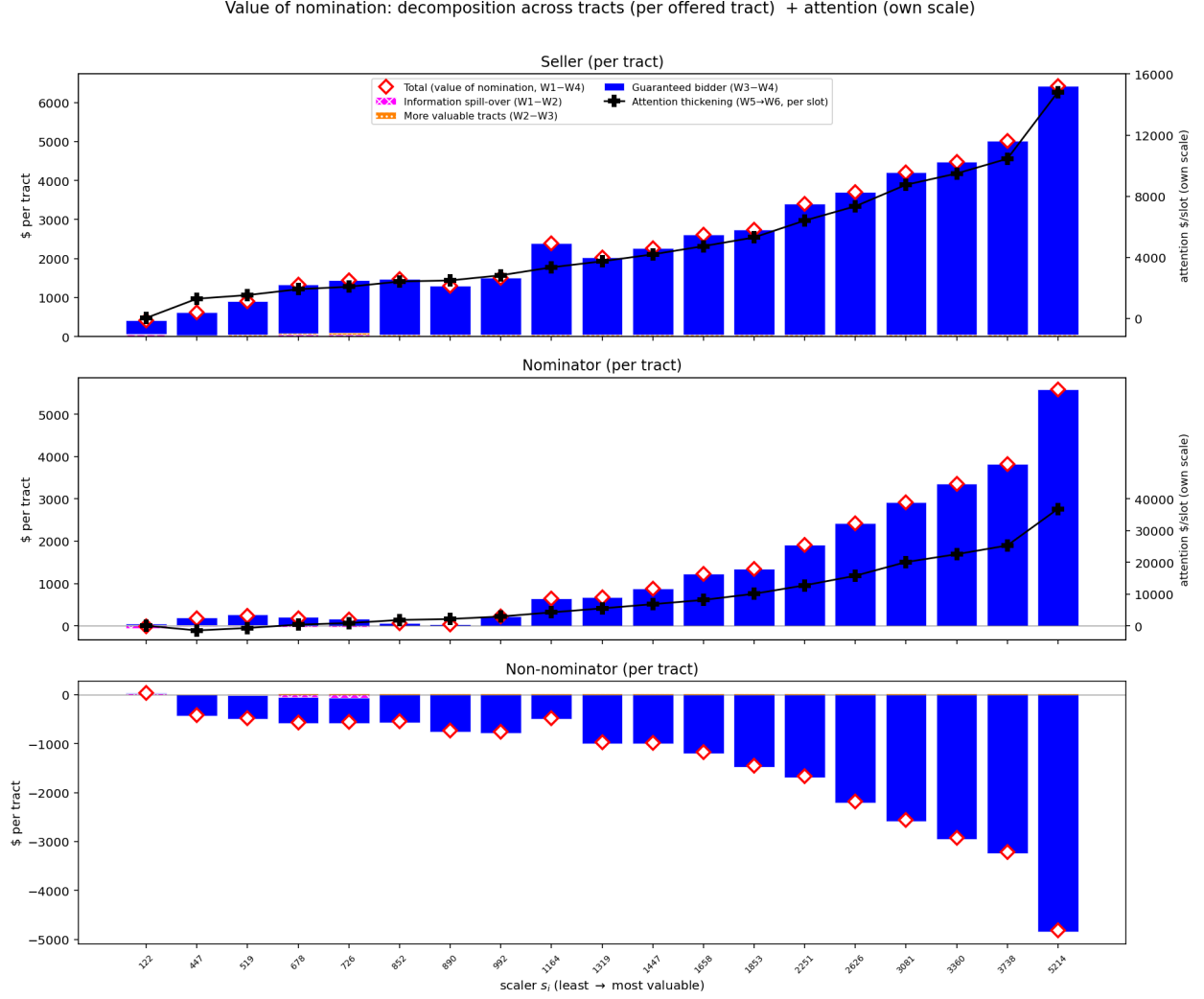


Figure 16: **Welfare across tracts (bins), by party.** Four-channel decomposition of the value of nomination per bin for the seller, nominator, and non-nominator, each **per offered tract** ($\mathcal{W}_4$ counts non-entered tracts as zero); the white/red diamond is the total before concentrated attention and the black “+” line is concentrated attention ($\mathcal{W}_5 \rightarrow \mathcal{W}_6$, per tract) on a zero-aligned secondary axis. Bins run from the least valuable tracts (Bin 19, left) to the most valuable (Bin 1, right).

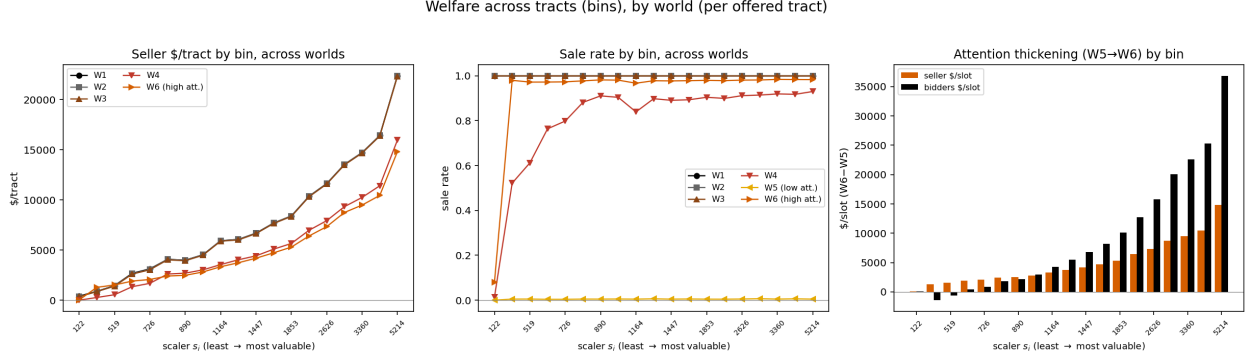


Figure 17: **Welfare across tracts, by world.** Seller revenue per offered auction and the sale rate in each counterfactual world, and concentrated attention ($\mathcal{W}_5 \rightarrow \mathcal{W}_6$), by bin. Under the thin fitted rate $\hat{\lambda}_b$ ($\mathcal{W}_5$) the sale rate is near zero except in the highest-attention bin; concentrating attention ($\mathcal{W}_6$) restores near-certain sale throughout. Bins run from the least valuable tracts (Bin 19, left) to the most valuable (Bin 1, right).

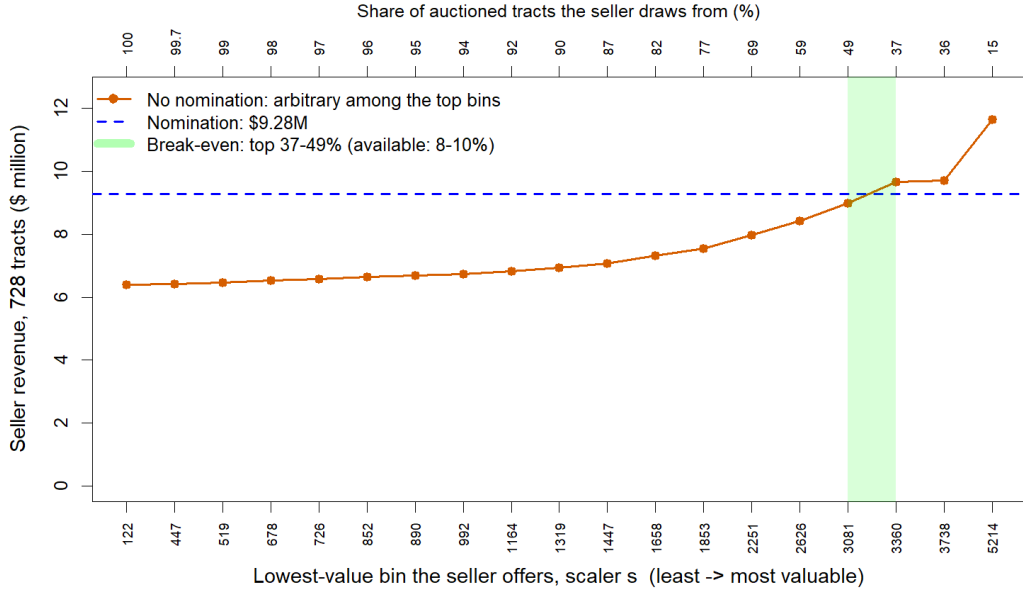


Figure 18: **How much would the seller need to know?** Seller revenue over 728 offered tracts when the seller runs no nomination and offers tracts arbitrarily from the top bins by observable value multiplier s_b , with the included bins in proportion to their auction shares, against revenue under nomination (dashed). The horizontal axis is the least promising bin the seller still offers, labelled by its multiplier s_b and running from the least to the most promising; the top axis gives the share of auctioned tracts in the included bins. Offering all bins (the leftmost point) reproduces the no-nomination benchmark $\mathcal{W}_4$. The shaded band marks the break-even: offering only Bins 1–3 (8% of available tracts; 37% of auctioned tracts) beats nomination, while offering Bins 1–4 (10% of available tracts; 49% of auctioned) does not. Each bin is valued at its median tract.

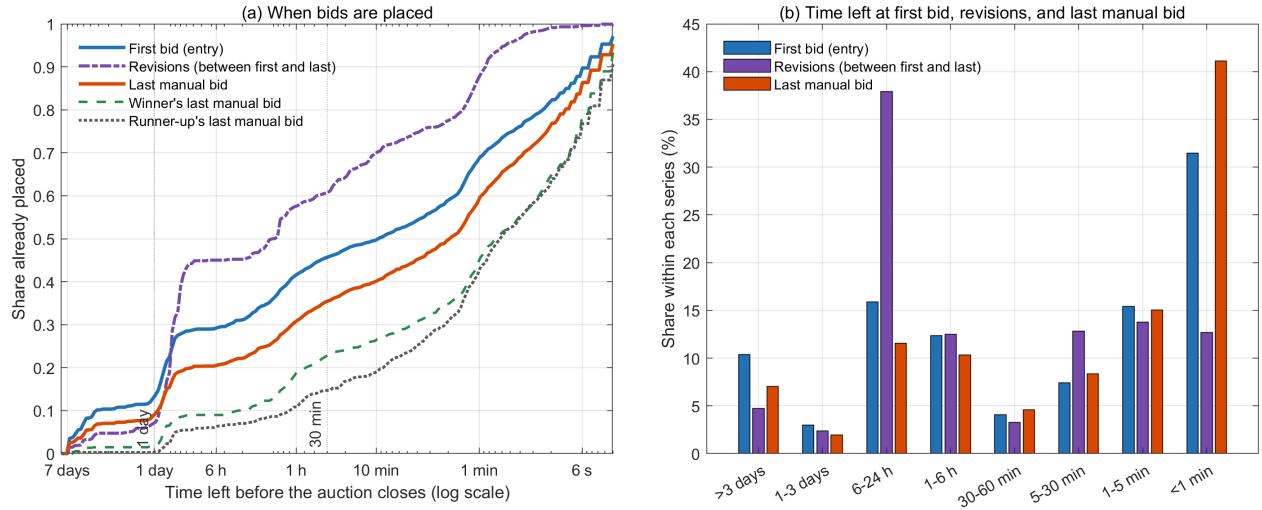


Figure 19: When Bids Are Placed

Notes: Panel (a): share of bidder-tract pairs whose first bid, last bid, winning bid, and runner-up bid have already been placed, and share of intermediate revisions already placed, as time runs from seven days before the close to the close (log scale). Panel (b): share of each series by time left before the close. 2,740 bidder-tract pairs and 5,001 bids.

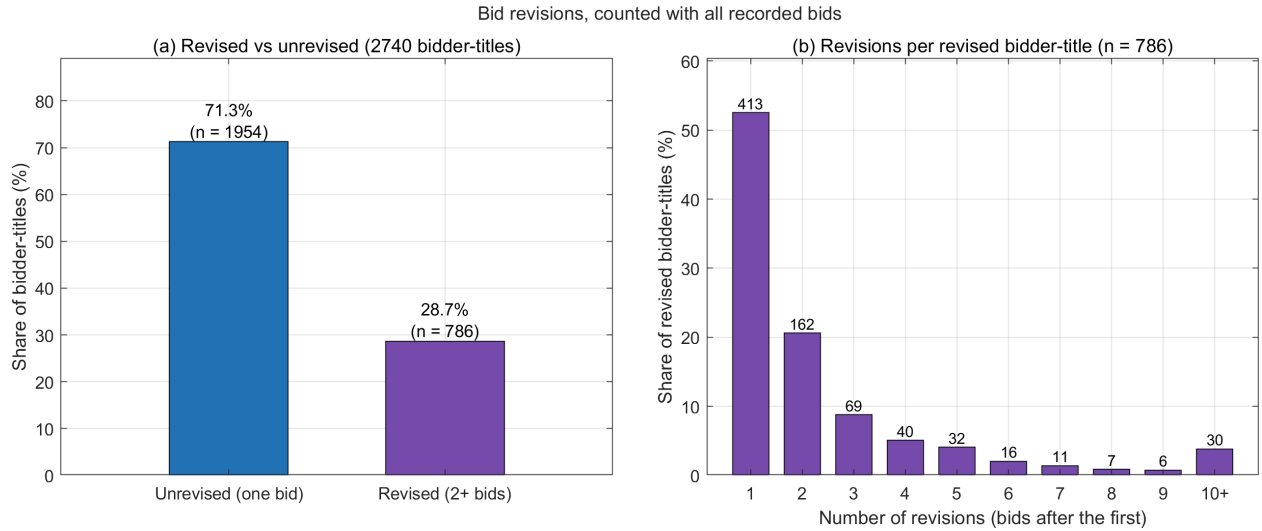


Figure 20: How Often Bids Are Revised

Notes: Panel (a): bidder-tract pairs with one bid (unrevised) and with two or more bids (revised). Panel (b): among revised pairs, the number of revisions (bids after the first), with ten or more pooled.

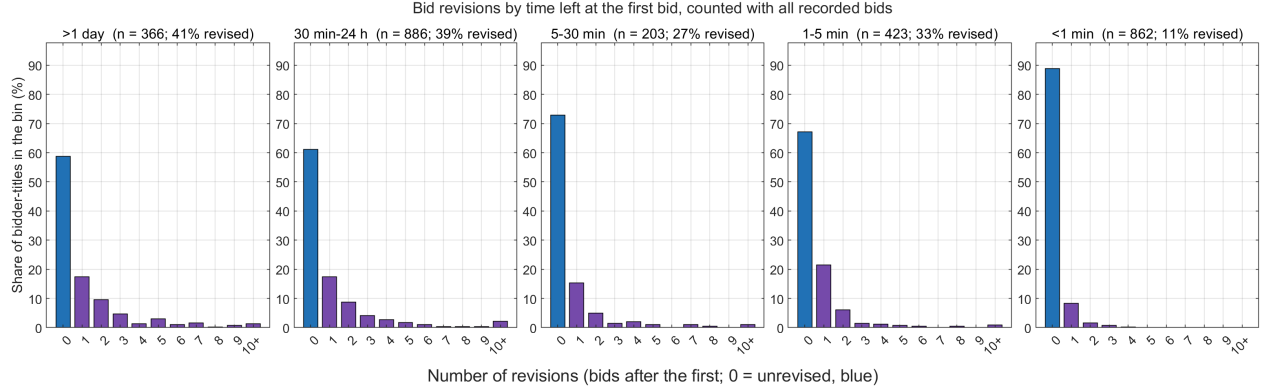


Figure 21: Revisions by When the Bidder First Bids

Notes: Each bidder–tract pair is grouped by the time left at its first bid. Bars give the share of pairs in the group with zero (unrevised), one, ..., nine, and ten or more revisions. Groups by time left at the first bid: more than one day, 30 minutes to 24 hours, 5–30 minutes, 1–5 minutes, and less than one minute.

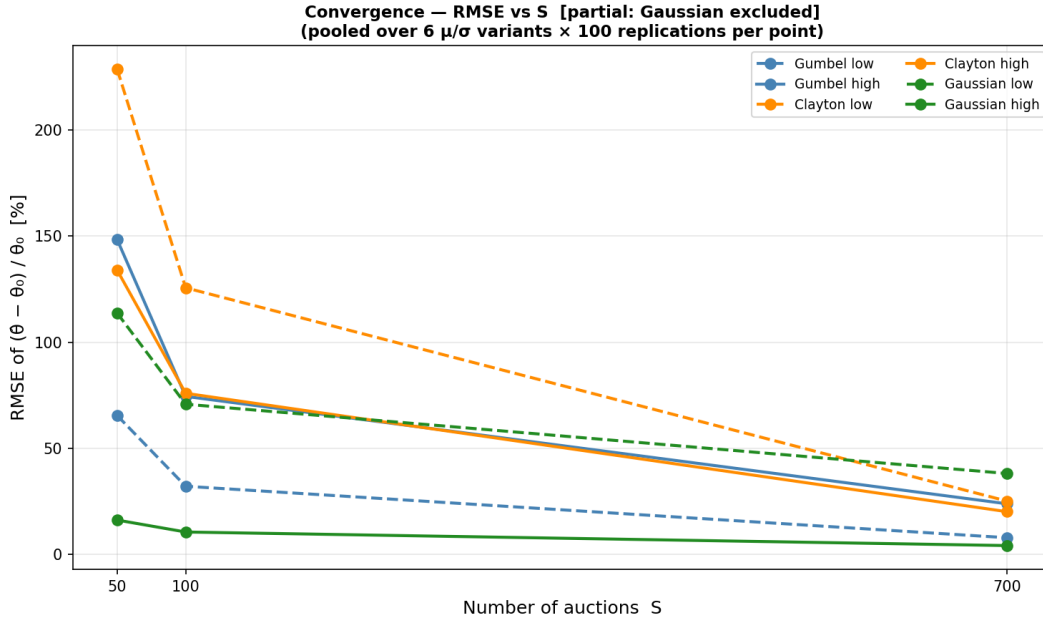


Figure 22: RMSE of the normalised deviation $(\hat{\rho} - \rho_0)/\rho_0$ versus number of auctions S , pooled over six lognormal marginal variants (600 replications per point). Solid lines = high dependence; dashed lines = low dependence. All copula families converge toward zero as S grows.

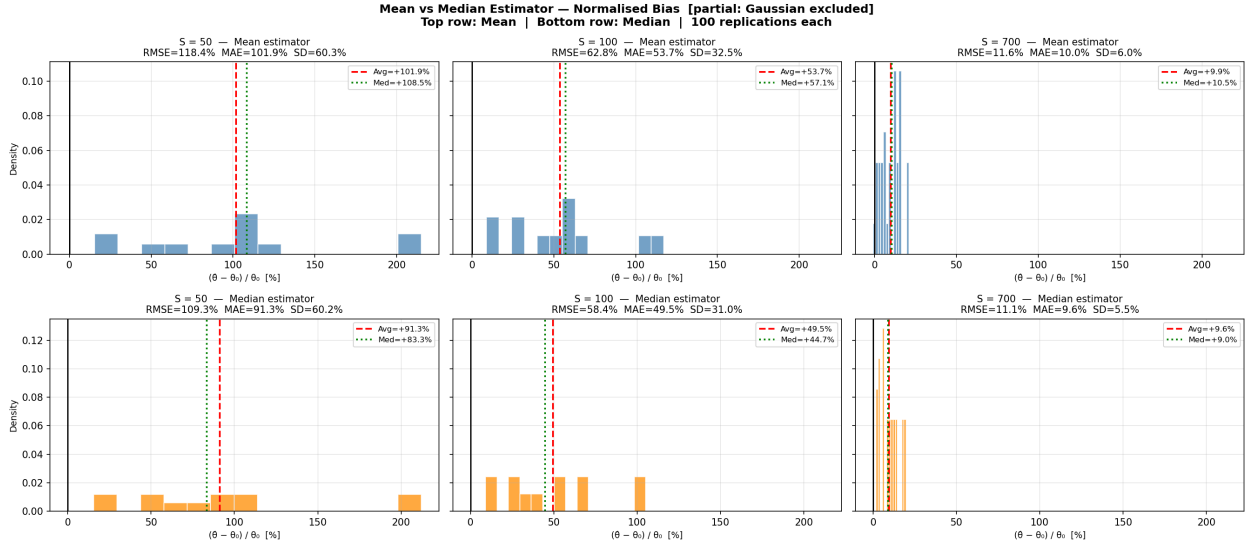
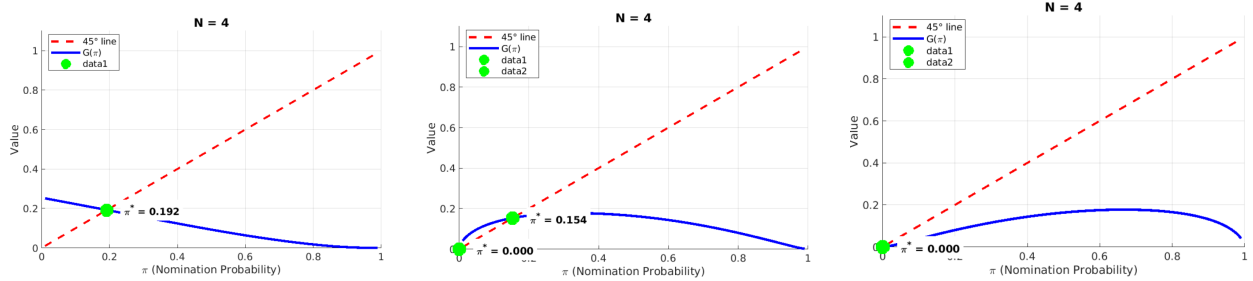


Figure 23: Distribution of the normalised deviation $(\hat{\rho} - \rho_0)/\rho_0$ across all 36 DGP cells, separately for the mean aggregator (top row) and the median aggregator (bottom row), at $S \in \{50, 100, 700\}$ (columns left to right). Red dashed line = average of the displayed distribution; green dotted line = median of the displayed distribution. The mean distribution has a heavier right tail at small S (driven by Gumbel and Clayton cells), which the median aggregator substantially dampens.



(a) $K = 1$: Unique equilibrium (b) $K = 2$: Multiple equilibria (c) $K = 3$: Coordination failure

Figure 24: Fixed Point Problem $G(\pi; K, I) = \pi$ for $I = 4$ Bidders (Simplified Model)

Each panel plots the operator $G(\pi; K, I)$ (blue curve) against the 45-degree line (red dashed). Equilibria correspond to intersections. Panel (a): strategic substitutes under $K = 1$, unique equilibrium. Panel (b): strategic complements under $K = 2$, multiple equilibria including coordination failure. Panel (c): $K = 3$, only coordination failure. This simplified model abstracts from value uncertainty and late learning.

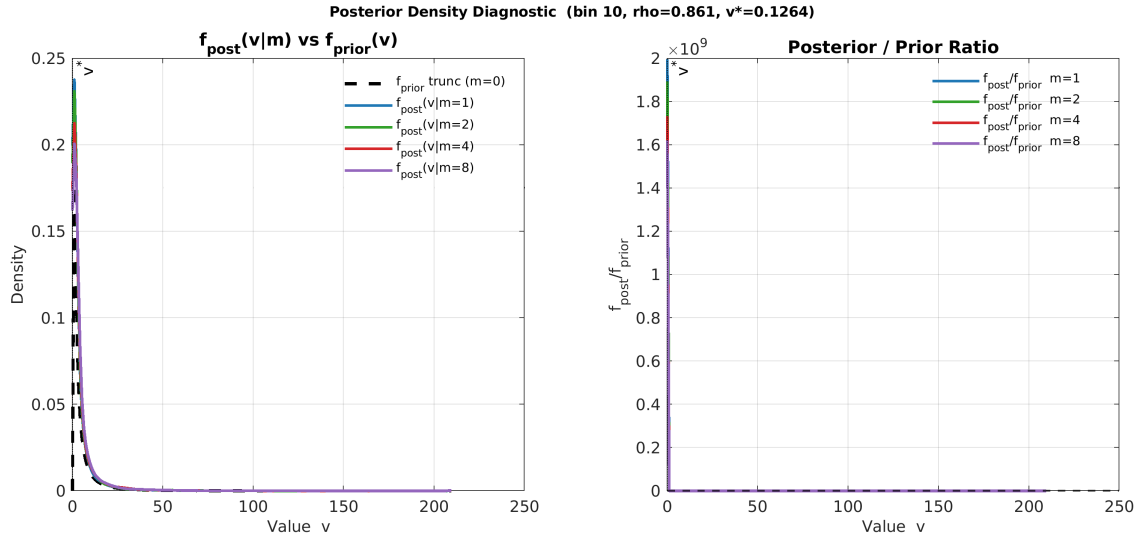


Figure 25: Posterior Density vs. Prior, and Their Ratio (Estimated Parameters)

Notes: Left panel: prior density truncated at v^* (black dashed, the $m = 0$ baseline) and copula posterior $f_{\text{post}}(v | m)$ at $m \in \{1, 2, 4, 8\}$ (colored solid). Right panel: point-wise ratio $f_{\text{post}}(v | m)/f_{\text{prior}}(v)$. Vertical dotted line marks v^* . Away from the boundary the ratio is indistinguishable from 1, so the conditional density shape is essentially unchanged by the Bayesian update at $u^* \approx 0$; the large spike near the lower boundary $v \approx v^*$ (of order 10^9) is a numerical artifact of dividing by the sharply truncated prior density there and carries negligible probability mass. Reported for Bin 10 (median bin), $\hat{\rho} = 0.861$, $v^* = 0.1264$ ($u^* \approx 0$).